Sources of lifetime inequality revisited

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October 30, 2019

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Abstract

Factors determined early in life can be key determinants of the lifetime value of earnings, consumption and wealth. Furthermore, some of these variables are determined by parental background (ability, human capital) or passed on directly from parents (wealth). In this paper, I study an OLG-life cycle economy with borrowing constraints and costly human capital acquisition, in which initial conditions are determined by parental background. The cost of human capital may prevent constrained agents to optimally acquire human capital and intergenerational transmission of wealth may alleviate this effect for wealthy households. Using PSID and NLSY data, I document statistics of the evolution of cohort inequality and the importance of parental transfer to attend college. The model suggests that initial wealth can explain about 20% of cohort income inequality. Relaxing borrowing constraints decreases the importance of initial wealth. Shutting down intergenerational correlation of abilities decreases the importance of initial conditions, suggesting an important role for parental background.

1 Introduction

How important are initial conditions to determine one’s lifetime welfare, in terms of consumption and income? Initial conditions, such as one’s cognitive ability or wealth can have a sizable impact on future income. High ability individuals are capable to increase their human capital faster and thus obtain higher future earnings. Financial wealth allows to smooth adverse economic shocks, to increase consumption level and even to get specific
skills by attending college. Furthermore, these initial conditions strongly depend on parental background. Abilities can be transmitted through genes. Financial wealth can be directly transmitted to descendants.

Some authors such as Piketty (2014) and others have argued that financial wealth has become more important to determine one’s lifetime outcomes. Others, such as Huggett, Ventura, and Yaron (2011) (HVY, hereafter) argue that financial wealth has little impact to determine one’s lifetime welfare and the observed inequality. To assess the importance of initial condition it is important to take into account both parental background and financial markets.

In this paper, I assess the impact of initial conditions on lifetime welfare. In particular, I study the role of parental background and wealth, and how these intergenerational links contribute to shaping the observed inequality.

I develop a quantitative model with a life cycle structure and altruistic agents. Agents are heterogeneous in three dimensions: wealth, innate ability, and human capital. Innate ability is fixed in a lifetime and correlated across generations. Individuals with high ability accumulate human capital faster. Agents go through three different stages: education, work, and retirement. Education allows them to acquire skills specific to more productive sectors, and as a result, they get a higher wage per unit of human capital provided in the market. In order to get education, they need financial resources obtained from parents. During the working stage agents save for retirement and to transfer to their kids. In retirement period agents live off a pension provided by the government and their own savings.

Using Data from the PSID I show the evolution of mean earnings and the evolution of inequality of a cohort during the life cycle. I also provide evidence of the probability of attending college conditional on parental education. Finally, I show evidence of the importance of wealth to attend college. In particular, data suggest that in recent years wealth is much more important than it used to be.

Using Data from the NLSY79 and NLSY79 Child and Young adults I show that the intergenerational correlation of abilities is between 0.3 and 0.4. Using Data from NLSY97 I show the importance of parental transfers in obtaining higher education. I show that the distribution of transfers is highly skewed. I estimate a logit model and show that the parental transfers are statistically significant, under several specifications.

The calibrated model suggests that parental wealth has a great impact on determining
lifetime welfare. An agent in the 75th percentile of the initial wealth distribution has 32% higher lifetime earnings than an agent with median wealth. Agents in the 75th percentile of the ability distribution have 5% higher earnings than the agent with the median ability. Variance decomposition analysis suggests that more than 20% of the observed cohort inequality in income and wealth is due to initial conditions.

Improving credit access decreases the importance of initial conditions. When agents are allowed to borrow as much as they need to attend college, the fraction of the variance of wealth and income explained by initial conditions decreases by a 20%.

Parental background is also important to explain the variation of income and wealth over the life cycle. Shutting down the intergenerational correlation of abilities decreases the fraction explained by initial conditions by roughly 20%.

The rest of the paper is organized as follows. Section 2 reviews the literature. Section 3 describes the life cycle model. Section 4 uses PSID and NLSY data to show the evolution over the life cycle of mean earnings and several measures of inequality for a particular cohort, as well as the importance of parental education, financial wealth and transfer in the process of human capital investment. Section 5 discusses the calibration and Section 6 discusses the model fit and performs quantitative exercises. Section 7 concludes.

2 Related Literature

The idea of intergenerational transmission of wealth from parents to children and human capital accumulation dates back to Loury (1981). In his model, parents are subject to borrowing constraints and therefore they may not be able to optimally invest in their child’s human capital. This mechanism endogenously generates the earnings distribution in the economy. Despite that this mechanism has been well-known for a long time, the quantitative impact of this mechanism has not been explored in quantitative macroeconomic models.

There are several papers studying intergenerational transmission of wealth, De Nardi (2004), Boar (2017), Luo (2017), to name a few. Most of them study bequests and/or inter vivos transfers, but none of them study the relationship between transfers and human capital accumulation.

There is a vast literature explaining earnings inequality during life cycle, for instance,

Daurich (2017) studies an economy with intervivos transfers and human capital accumulation, where parents influence their child’s human capital by investing time and money. His focus is different in that he studies the macroeconomic implication of changing schooling financing by changing the tax policy, whereas my focus is to quantitatively disentangle the effects of initial conditions, namely learning ability, initial human capital and initial wealth, in order to explain the lifetime inequality.

Credit constraints and human capital acquisition have been study in Lochner and Monje-Naranjo (2011) and Hai and Heckman (2017). They focus in the interaction of credit constraints human capital rather than their implications for inequality and welfare.

Abbott et. al. (2018) study an economy where financial frictions play a role in human capital acquisition. Their main focus is the implication of education policy in determining intervivos transfers.

The relative importance of shocks versus initial conditions in order to explain lifetime welfare has been studied in Huggett, Ventura and Yaron (2006) and Huggett, Ventura and Yaron (2011). My paper contributes to this literature by reassessing the relative contribution of each factor in an economy with intergenerational links across generations and costly human capital accumulation.

3 Economic Environment

Demographics: Life cycle model with overlapping generation structure and deterministic finite lives. At age 30 agents have a child. Agents die for sure at age 80.

Heterogeneity: Agents are heterogeneous in innate ability \( \theta \), human capital \( h \) and financial assets \( a \). Innate ability is determined at the time of birth and is fixed throughout the agent’s lifetime. The initial level of human capital (at age 18) is determined by innate ability. I assume that there is an initial level of human capital \( h_0 \) that agents are born with. During the first 18 years of life, agents devote all their time endowment to accumulate human capital. Differences in innate ability generate differences in initial human capital. This mechanism generates a positive correlation between initial human capital and innate
ability. Ability allows agents to acquire human capital faster, so it can be interpreted as learning ability. Agents have two sources of human capital accumulation (i) acquire formal education (going to college) (ii) acquire human capital spending time during their working life (e.g. on the job training), à la Ben-Porath. During the working stage agents receive i.i.d. idiosyncratic shocks to human capital, \( z_t \). Financial assets are accumulated through savings during the lifetime.

**Life cycle:** Agents go through three stages in their lives: (i) education, (ii) labor and (iii) retirement. In the first stage of their lives, agents decide whether to go to college. This decision depends on their innate ability, initial human capital as well as the availability of financial resources. If they choose to go to college, agents acquire high skill human capital. Otherwise, they stay as low skilled workers. Being high skilled worker can be interpreted as working in certain “occupations”, “industries” or “sectors” that pay higher wage rates per unit of human capital.

During their labor stage, agents decide how much time spend on labor activities and how much time to spend acquiring human capital. Human capital is affected by individual investments as well as stochastic shocks. By age 30, the individual’s child is born. By age 48 the child becomes an adult and his parent must decide how much financial resources to transfer him. During retirement, agents receive social security benefits and no labor earnings. There are no bequests, so agents consume all their wealth at the end of their lives.

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1. Huggett et al. (2011) show that to replicate the increasing inequality over the life cycle a positive correlation between ability and initial human capital is necessary. Guvenen et al. (2013) offer a rationale for this condition. Since agents enter the model at some point in their lifetime, they have already accumulated some human capital in early stages in life. Thus, agents with higher ability accumulate more human capital which generates a positive correlation. This is the exact mechanism used in the periods before the model starts.

2. It is assumed that once the wealth transfer is made, the child is free to spend those resources in education or consumption. Parents do not have the technology to monitor how kids spend resources nor can make contingent transfers.
Figure 1: Timeline

Figure 1 shows the timeline of the model.

**Intergenerational Links:** Innate ability is correlated across generations and follows a process as

\[
\log(\theta_{n+1}) = (1 - \rho) \log(\bar{\theta}) + \rho \log(\theta_n) + \varepsilon
\]  

where \(\theta_n\) is the ability of generation \(n\), \(\bar{\theta}\) is the log run average of ability, \(\rho\) is the intergenerational correlation of ability and \(\varepsilon\) is an i.i.d. shock with mean zero and variance \(\sigma_\varepsilon\). The intergenerational correlation of abilities generates an intergenerational correlation of initial human capital. Wealth is transmitted across generations only through the intervivos transfer, no bequests. Parents have a preference shock \(\epsilon_1, \epsilon_2\) over child’s college decision, where \(\epsilon_1, \epsilon_2\) are the preference shock of not attending college and that of attending college, respectively.

**Borrowing constraints:** In order to fully assess the contribution of borrowing constraint to human capital accumulation I use different borrowing limits: (i) no borrowing is allowed, (ii) ad-hoc borrowing limits.

**Preferences:** Agents have altruistic preferences towards their children. Let \(U_n\) be the lifetime utility derived from consumption flow for agent of generation \(n\), \(U_n = \sum_{t=1}^{T} \beta^t u(c_t)\), where \(u(c_t)\) is increasing and concave function. Individuals maximize the lifetime utility from their own consumption and weight their child’s lifetime utility of consumption. They maximize \(U_n = U_n + \tilde{\gamma} U_{n+1}\), where \(\tilde{\gamma}\) is the weight parents put on their kid’s utility.

**Tax schedule and Government spending:** The tax and transfer system has two components: an income tax component and a social security system component \(T = T^{ss} + T^{inc}\). The social security part consists of a proportional tax \(\tau^{ss}\) for active workers, and a positive
transfer equal to a fraction of average earnings in the economy, as in Huggett et al. (2011). The income tax schedule captures the effective average income tax rates of the economy as a function of income. In particular, tax rate function \( t(\tilde{y}) = 1 - \mu \tilde{y}^{-\tau} \), as in Bernabou (2002) and Heathcote et al. (2014), where \( \tilde{y} \) is taxable income. I consider taxable income as the after social security tax labor income plus capital income. Guner et al. (2014) show that this tax function specification provides a reasonable fit to the actual U.S. income tax schedule for both the average tax rates and effective marginal tax rates. The government has a balanced budget every period.

3.1 The Life cycle model

Stage 1: Education stage

Agents choose whether to go to college or not. The value of an agent at time zero, with initial financial wealth \( a \), human capital \( h \) and ability \( \theta \) is:

\[
V_0(a, h; \theta) = \max \{V^c(a, h; \theta), V^{nc}(a, h; \theta)\}
\]

where \( V^c(k, h; a) \) is the value of a college student and \( V^{nc} \) is the value of not going to college.

The college student solves

\[
V_j(a_j, h_j; \theta) = \max_{c_j, s_j} u(c_j) + \beta \mathbb{E}_z[V_{j+1}(a_{j+1}, h_{j+1}; \theta)]
\]

s.t.
\[
\begin{align*}
    c_j + a_{j+1} &= a_j(1 + r) - P_c \\
    h_{j+1} &= ((1 - \delta)h_j + \theta(h_j)\alpha) \\
    a_{j+1} &\geq a_{j+1}
\end{align*}
\]

If the agent decides to attend college, she spends all her time endowment accumulating human capital. Furthermore, there are no human capital shocks during college. A college graduate works at a wage of \( w_H \) per unit of human capital after the education stage.

If the agent chooses not to go to college, she starts working as a low skilled worker and solves
\[ V_j(a_j, h_j; \theta) = \max_{c_j, s_j} u(c_j) + \beta \mathbb{E}_z[V_{j+1}(a_{j+1}, h_{j+1}; \theta)] \]

s.t. \[ c_j + a_{j+1} = a_j (1 + r) a_j + \mu (r a_j + (1 - \tau_{ss}) w_L h_j(1 - s_j))^{1 - \tau} \]
\[ h_{j+1} = \exp(z_{j+1})((1 - \delta) h_j + \theta(s_j h_j)^{\alpha}) \]
\[ a_{j+1} \geq a_{j+1} \]

Stage 2: Working stage

Before Transfer

Agents work either as a high skilled worker or low skilled worker, depending on whether they go to college. High skilled workers receive a wage of \( w_H \) per unit of time and human capital, whereas low skilled workers receive \( w_L \).

During this stage, agents have incentives to save for three motives: (i) self-insurance and smooth consumption, (ii) transfer wealth to kids, (iii) save for retirement.

The value function of a worker hired at a wage rate \( w \), with assets \( a \), human capital \( h \) and ability \( \theta \) is

\[ V_j(a_j, h_j; \theta) = \max_{c_j, s_j} u(c_j) + \beta \mathbb{E}_z[V_{j+1}(a_{j+1}, h_{j+1}; \theta)] \]

s.t. \[ c_j + a_{j+1} = a_j (1 + r) a_j + \mu (r a_j + (1 - \tau_{ss}) w_L h_j(1 - s_j))^{1 - \tau} \]
\[ h_{j+1} = \exp(z_{j+1})((1 - \delta) h_j + \theta(s_j h_j)^{\alpha}) \]
\[ a_{j+1} \geq a_{j+1} \]

Transfer

At age 48, when the individual’s child becomes an adult, the parent transfers resources to the kid. At this age, the value function is

\[ V_j(a_j, h_j; \theta) = \max_{c_j, s_j, tr} u(c_j) + \beta \mathbb{E}_z[V_{j+1}(a_{j+1}, h_{j+1}; \theta)] + \gamma \mathbb{E}_{\theta'}[V(tr; \theta')] \]

s.t. \[ c_j + a_{j+1} + tr = a_j (1 + r) a_j + \mu (r a_j + (1 - \tau_{ss}) w_L h_j(1 - s_j))^{1 - \tau} \]
\[ h_{j+1} = \exp(z_{j+1})((1 - \delta) h_j + \theta(s_j h_j)^{\alpha}) \]
\[ a_{j+1} \geq a_{j+1} \]
where \( \mathbb{E}_{\theta'}[V(tr; \theta')] \) is the expected value of lifetime utility of the son with initial wealth \( tr \). The expectation operator is over the child’s ability \( \theta' \) and the parent’s preference shocks.

**After transfer**

Once the transfer has been made, agents accumulate wealth only for retirement and smooth consumption. They solve:

\[
V_j(a_j, h_j; \theta) = \max_{c_j, s_j} u(c_j) + \beta \mathbb{E}_z[V_{j+1}(a_{j+1}, h_{j+1}; \theta)]
\]

s.t. \( c_j + a_{j+1} = a_j (1 + r) + \mu (ra_j + (1 - \tau_{ss})w_l h_j (1 - s_j))^{1-\gamma} \)
\[
h_{j+1} = \exp(z_{j+1})((1 - \delta)h_j + \theta(h_j)^{\alpha})
\]
\[
a_{j+1} \geq a_{j+1}
\]

**Stage 3: Retirement**

Retirees live off their wealth and receive a constant pension benefit \( p \). The retirees’ problem is

\[
V(a_j) = \max_{c_j} u(c_j) + \beta V(a_{j+1})
\]

s.t. \( c_j + a_{j+1} = p + (1 + r)a_j \)

### 3.2 Equilibrium

A Steady State Competitive Equilibrium is a set of policy functions \( \{c_t, s_t\}_{j=1} \), value functions \( \{V_t(s; \theta)\}_{j=1} \) and distribution over states \( \{g_t(s, \theta)\}_{j=1} \) such that given factor prices \( r, w_L, w_H \), government spending and tax schedule \( \{T, G\} \) and initial distribution over states \( g_0(s, \theta) \):

1. Given prices and government policies, policy functions and value functions solve the individual’s problem.
2. Distributions are consistent with policy functions and initial distribution.
4 Empirical Analysis

In this section, I start describing the data sources and the sample selection criteria. Then I report several facts. First, I document mean income profiles and inequality measures by cohort, in a similar fashion as in HVY. Next, I document income profiles by education. Then I document the persistence of education levels across generations. Afterward, I document the levels of educational attainment by wealth class. Finally, I document the probability of attending college conditional on parental transfers.

4.1 Data sources

The Panel Study of Income Dynamics started in 1968 and surveys more than 18,000 individuals from 5,000 families and over five generations, representative of the United States. Annual waves up to 1997 and biannually since then. It collects data on family and individuals, with more details on household head and wife. After a child leaves home and forms his own household, his family is also part of the survey. I use mainly information on the household head.

The National Longitudinal Survey of Youth (NLSY) surveys individuals aged 14 to 22 at the time of their first interview. There are two different cohorts, NLSY79 which is first interviewed in 1979 and NLSY97 which interviews youths for the first time in 1997. After the first interview, they follow the cohort with annual or biannual surveys. NLSY79 and NSL97 have a sample size of around 12,000 and 9,000 individuals approximately, respectively. I use information from these cohorts to compute information on earnings as well as on parental transfer during young ages.

4.2 Income profiles in the PSID

To compute income profiles I focus on male household heads from the SCR sample (nationally representative). For individuals aged less or equal than 30, they have to work more than 260 hours a year and earn at least $1,000 a year (1970 dollars). Individuals older than 30 are kept in the sample if they earn at least $1,500 and work more than 520 hours. To construct statistics, I consider a 5-year bin for each age.

Ideally, I’d like to estimate the age profile for several statistics by estimating the following regression
\[ \text{stat}_{i,j} = \text{age effect}_i + \text{time effect}_j + \text{cohort effect}_c + \varepsilon_{i,j} \quad (2) \]

where \( \text{stat}_i \) is the statistics of interest for age \( i \).

To estimate this equation I can control for either time fixed effect or cohort fixed effect. Controlling for both is not possible since age, time and year are collinear. I show results by controlling for one of these effects at a time. For a discussion on the differences between controlling for time or cohort effects see Heathcote et al. (2005).

Panel A in Figure 2 shows the mean earnings age profile. The differences between cohort and time effect are not noticeable until the end of the life cycle. The figure displays an increase and concave pattern, with a small decreasing pattern at the end when controlling for time effect.

Panel B in Figure 2 displays the evolution of cohort earnings inequality as the cohort ages. The pattern is increasing. Unlike the earnings profile, the shape of the Gini coefficient profile depends crucially on whether we control for time or cohort effect.
4.3 Incomes profiles by education

Now I focus on just mean earnings, and compute the same statistical model as (2) with two different subsamples: one with college-educated individuals and one with the level of education less or equal than high school graduate. I use the same criteria as the previous section to select the sample.

Figure 3 shows the income profile for college-educated and non-college educated workers using PSID data. In the appendix, I show that a similar earnings age profile is observed in the NLSY data.

Earnings profiles differ by education attainment. Earnings profile for the college-educated workers is steeper and reaches a higher level at the end of the life cycle than that of high school-educated workers. Guvenen (2009) documents heterogeneous income profiles. Figure 3 is consistent with that view.

4.4 Intergenerational correlation of abilities

Using data from the NLSY79 and the NLSY79-Children and Young adults, I merge information from mothers and children to explore how cognitive abilities are transmitted across
generations. I use the AFQT percentile score as a measure of ability for parents and PIAT scores for children.

![Intergenerational correlation of abilities](image)

Figure 4: Intergenerational correlation of abilities, math scores

Figure (4) shows the relationship between mothers’ AFQT scores and children’s math PIAT scores. The correlation between mother’s and children’s ability is between 0.3 and 0.4, depending on whether I use math, reading recognition or reading comprehension as a measure of ability for children. This correlation is significant in all of these cases.

### 4.5 Intergenerational persistence of education

I compute several statistics on the persistence of education across generations. I consider waves from the PSID from 1974-2015 because there is enough information about the mother’s and father’s education. I focus on the group of 23-27 years old (25 years old bin).

I compute the probability of attending college, conditional on parental education. I condition in father’s attending college, mother’s attending college, and both parents college-educated. I repeat the exercise when parents do not go to college.
Figure 5: Probability of being college educated, conditional on parent’s education

Figure 5 shows the probability of going to college conditional on parent’s education. Panel A shows that conditional on both parents being college-educated, the probability of being college-educated is higher than the probability when only one parent went to college, for all years. Panel B shows conditional on both parents being non-college educated, the probability of being college-educated is also higher, although quantitatively small, than the probability when conditioning in only one parent being non-college educated. Both graphs display an upward trend, which represents an increase in enrollment in college education. There are not quantitatively important differences when doing the analysis by gender (shown in the appendix). On average, the probability of attending college conditional on parents being college educated is 0.5, and the probability when parents do not have a college education is 0.2.

4.6 Education by wealth class

In this section, I explore the conditional probability of attending college, given household wealth. Again, I focus on individuals aged 23-27. I constructed a measure of wealth class, according to the wealth distribution among those individuals. Each wealth class has the same size. Wealth classes 1, 2 and 3 correspond to the bottom, medium and upper side of the wealth distribution. I use wealth data from PSID, waves 1984, 1989, 1994, and the biannual waves from 1999 to 2015.
Figure 6: Probability of being college educated, conditional on household wealth

Figure 6 shows the probability of being college educated-conditional on wealth class. The probability of being college-educated conditional on being in the upper wealth class is higher for every year available. In recent years, the probability of attending college condition on being in the wealthy class has skyrocketed, from 27% in 1994 up to more than 50% in recent years, which suggest that wealth is more important to access to higher education in recent years. The conditional probability of going to college does not differ much for the bottom two wealth classes.

This analysis focuses on household wealth, and not on transferred wealth. At young ages, however, it is most likely that a great fraction of wealth corresponds to transferred wealth rather than self-made wealth. To better assess the contribution of transfers, in the next section I focus on whether the household has received gifts in recent years and its impact on education.

4.7 Education and parental transfers: Evidence from NLSY97

Because PSID lacks information on wealth transfers at young ages, I now use data from the National Longitudinal Survey of Youth, cohort 1997. The NLSY contains information on
young individuals aged 12-17 at the time of their first interview in 1997 (cohorts 1980-1984).

I focus on cohorts 1980 and 1981 because the information about transfers is more accurate and consistent across years\textsuperscript{3}. I construct a variable with the sum of parental transfers to the child, excluding allowances, from ages 16 to 22.

<table>
<thead>
<tr>
<th>Cohort</th>
<th>Q1</th>
<th>Median</th>
<th>Mean</th>
<th>Q3</th>
<th>Fraction Tr &gt;0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980</td>
<td>300</td>
<td>750</td>
<td>3493</td>
<td>2200</td>
<td>39.4%</td>
</tr>
<tr>
<td>1981</td>
<td>300</td>
<td>850</td>
<td>3292</td>
<td>2138</td>
<td>39.9%</td>
</tr>
</tbody>
</table>

Table 1: Descriptive statistics transfers

Table 1 shows some basic descriptive statistics about total transfers for cohorts 1980 and 1981. The histograms do not differ much. The distribution of transfer is extremely skewed, almost 40% receive no transfer, and the mean is more than three times the median. The maximum total transfer for each year is about 80,000.

\textsuperscript{3}During the first years of the survey, respondents give information about transfers received from mother and father, separately. If they do not provide an exact number, they can give an estimate. This information is enough to construct a reasonable estimate of parental transfers during ages 16-22. For later cohorts, the survey changes slightly, and it is impossible to identify the sources of transfer (they asked from transfers from other relatives as well). The exact amount of transfer is also lost, and there are only estimates of transfers. Transfers for Cohorts 1980 and 1981 rely less on those estimates.
Figure 7: Histogram of positive total transfers

Figure 7 shows the histogram for positive transfers for both years. I trim the scale to display the histogram appropriately. The histogram displays a high positive skewness.

I estimate several specifications of a logit model to assess the importance of transfers in the likelihood of attending college. Since some amount of the total transfer is imputed, I also consider a dummy variable of whether the respondent received a transfer.

Table 2 summarizes the results of a logit probability model with a dummy variable of being college-educated as independent variable. To control for transfers, I use both a continuous variable as well as a dummy variable. I also control for demographic characteristics, income and wealth and parental education. In all specification total transfers is statistically significant with 1 percent of confidence. In the appendix, I show that I obtain similar results when I estimate a linear probability model.

To interpret the coefficients, I compute the Average Marginal Effect (AME) and the Marginal Effect at Means (MEM) from all specifications with the dummy variable of transfers.

Table 3 shows that in every specification, the average marginal effect and the marginal effect at the mean go from 15% up to 20%. This means that, on average, there is a 20% extra chance of going to college for someone who has transfers, everything else constant.
### Table 2: Logit model

<table>
<thead>
<tr>
<th>Dependent variable:</th>
<th>educ</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td>log(1 + total_transfer)</td>
<td>0.170***</td>
</tr>
<tr>
<td>gender</td>
<td>0.428***</td>
</tr>
<tr>
<td>race</td>
<td>0.232***</td>
</tr>
<tr>
<td>log(fam_income)</td>
<td>0.472***</td>
</tr>
<tr>
<td>fam_wealth</td>
<td>0.003***</td>
</tr>
<tr>
<td>college_res_p</td>
<td>0.872***</td>
</tr>
<tr>
<td>received</td>
<td>1.019***</td>
</tr>
<tr>
<td>Constant</td>
<td>-1.351***</td>
</tr>
</tbody>
</table>

Observations: 1,867 1,867 1,635 1,139 1,867 1,867 1,635 1,139
Akaike Inf. Crit: 2,156.417 2,123.423 1,712.806 1,252.469 2,196.087 2,158.428 1,730.695 1,267.712

Note: *p<0.1; **p<0.05; ***p<0.01

### Table 3: Marginal Effects

<table>
<thead>
<tr>
<th>Specifications</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AME</td>
<td>0.217</td>
<td>0.207</td>
<td>0.140</td>
<td>0.1762</td>
</tr>
<tr>
<td>MEM</td>
<td>0.217</td>
<td>0.210</td>
<td>0.154</td>
<td>0.2015</td>
</tr>
</tbody>
</table>

Table 3: Marginal Effects
This is roughly in line with the estimates shown in the linear probability model discussed in the appendix.

5 Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Meaning</th>
<th>Value</th>
<th>Source/Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r )</td>
<td>rate of return</td>
<td>4%</td>
<td>calibrated</td>
</tr>
<tr>
<td>( w_l )</td>
<td>low wage</td>
<td>1</td>
<td>normalization</td>
</tr>
<tr>
<td>( w_h )</td>
<td>high wage</td>
<td>1.6</td>
<td>(Goldin and Katz (2007))</td>
</tr>
<tr>
<td>( p )</td>
<td>pension</td>
<td>15</td>
<td>40% of mean earnings</td>
</tr>
<tr>
<td>( P_c )</td>
<td>price of college</td>
<td>25</td>
<td>share college educated agents (30%)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Preferences and human capital</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
</tr>
<tr>
<td>( \sigma )</td>
</tr>
<tr>
<td>( \alpha )</td>
</tr>
<tr>
<td>( \gamma )</td>
</tr>
<tr>
<td>( \delta )</td>
</tr>
<tr>
<td>( \sigma_z )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Government</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu )</td>
</tr>
<tr>
<td>( \tau )</td>
</tr>
<tr>
<td>( \tau_{ss} )</td>
</tr>
</tbody>
</table>

Table 4: Baseline Calibration

Table 4 reports the parameters used in the solution of the model. The discount factor \( \beta \) is set to 0.95, a common value used in the literature for an annual discount factor. The risk aversion parameter \( \sigma \) is set to 2. The interest rate is set to 4%. The wage for the low wage sector is normalized to 1. The wage in the high wage sector is set to 1.6, that is, there is a 60% skill premium, as documented by Goldin and Katz (2007). The concavity of the human capital production function is set to 0.5, in line with estimates shown in the literature Huggett et al. (2011) and Guvenen et al. (2013). The pension benefit and the price of college are calibrated to target a 40% replacement rate and a 30% of college-educated agents. The tax schedule is taken from Guner et al. (2014) who show that the tax function in this paper is a good approximation of the US tax scheme. The social security tax rate is 10.6%, as in Huggett et al. (2011). The parameters of the initial distribution, as well as the parameter for altruism and human capital shocks are calibrated to match properties of the evolution of cohort inequality over the life cycle.
6 Analysis

6.1 Model Fit

Figure 8 shows the income profile from the baseline model. It resembles the increasing and concave pattern observed in the data.

![Image of Income profile](image1)

Figure 8: Income profile

![Image of Income profile by education](image2)

Figure 9: Income profile by education
Figure 9 shows the income profile by education groups. It shows a similar pattern as 3.

Figure 10 shows the evolution of the Gini coefficient over the life cycle. It displays a small increase over the life cycle, as suggested by the time effect Gini estimates shown in 2. The level is lower than the data.

Figure 10: Cohort Gini index over the life cycle

Figure 11: Initial wealth - parental transfers

Figure 11 shows the initial wealth distribution generated by the model. It is highly
skewed consistent with the empirical evidence. 51% of agents receive positive transfers. This is close to the empirical observation of 40%. The fraction of college-educated agents in the model is 45%.

6.2 Sources of Lifetime Inequality

In this section I compare the present discounted value of income, wealth and utility by changing 1) the level initial wealth, 2) innate ability. Table 5 summarizes the findings. I consider changes with respect to the median. For ability, I consider the 30th and 70th percentile \((\theta_l, \theta_h)\). For initial wealth, I consider the 75th percentile \(a_h\). Values expressed as variations with respect to the respective value evaluated at the median \((a_m, \theta_m)\).

<table>
<thead>
<tr>
<th>((a, h, \theta))</th>
<th>Lifetime utility</th>
<th>Lifetime earnings</th>
<th>Lifetime wealth</th>
</tr>
</thead>
<tbody>
<tr>
<td>((a_m, \theta_l))</td>
<td>-4.08%</td>
<td>-7.8%</td>
<td>-7.74%</td>
</tr>
<tr>
<td>((a_m, \theta_h))</td>
<td>5.57%</td>
<td>10.3%</td>
<td>10.2%</td>
</tr>
<tr>
<td>((a_l, \theta_m))</td>
<td>-1.82%</td>
<td>-0.8%</td>
<td>-1.6%</td>
</tr>
<tr>
<td>((a_h, \theta_m))</td>
<td>6.56%</td>
<td>0.5%</td>
<td>4.52%</td>
</tr>
</tbody>
</table>

Table 5: Lifetime Inequality

Table 5 shows the changes in lifetime utility, earnings and wealth when there is a change in initial conditions. Changes in wealth and in initial human capital have a great impact in lifetime welfare. The effect of ability is lower. These results are in stark contrast with Huggett et al. (2011), who argue that among initial conditions, ability and human capital have a great impact and that of initial wealth was almost negligible. There are two reasons that explain the different findings. First, they do not take into account that financial wealth is necessary to acquire specific human capital. For some agents, a small extra human capital has a big impact since they are now able to go to college. Second, they do not consider borrowing constraints. In the baseline model, agents are not allowed to borrow to attend education. In the next section, I explore the effect of having different borrowing constraints on lifetime welfare.

To get a better understanding of how initial conditions matter, I plot the changes in utility, earnings and wealth as we move along the ability and wealth distributions. Figure

---

4Utility changes are expressed as consumption equivalent variations
12 shows how different measures of welfare change, presented discounted value of utility, income and wealth, as we move along the distribution of abilities, in comparison to the mean values of each one of these measures of welfare. By construction, each graph crosses the y axis at 0 at the median of the distribution. They all show a similar pattern, as we move high along the distribution, utility is increasing and has a convex shape at the very left end.

Figure 13 shows the result of the same exercise as in Figure 12, but using the distribution of initial wealth. Welfare gains increase much more at the top of the distribution. In terms of income, the discontinuity is due to the fact that near that level of wealth, most agents decide to go to college, and then they get a higher wage per unit of human capital.

Figure 12: Change in utility, earnings and wealth as function of ability

Figure 13: Change in utility, earnings and wealth as function of initial wealth

6.3 Variance decomposition

In this section, I want to study the variance decomposition of lifetime income, lifetime wealth and lifetime utility. I want to decompose the variance in two: variance associated
with differences in initial condition vs variance associated with differences in shocks received during the lifetime.

For a variable $Y$ (lifetime income, lifetime wealth, lifetime utility), I can use the following decomposition

$$\text{var}(Y) = \mathbb{E}(\text{var}(Y|X)) + \text{var}(\mathbb{E}(Y|X))$$

(3)

where $X$ is a vector of initial conditions. In appendix, I show how to compute each of these terms in a dynamic programming problem.

<table>
<thead>
<tr>
<th>$Y$</th>
<th>percentage of variance due to initial conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lifetime income</td>
<td>4.78%</td>
</tr>
<tr>
<td>Lifetime wealth</td>
<td>8.85%</td>
</tr>
<tr>
<td>Lifetime utility</td>
<td>4.00%</td>
</tr>
</tbody>
</table>

Table 6: Variance decomposition

Table 6 shows the fraction of variance in lifetime utility, lifetime earnings and lifetime wealth associated with initial conditions. APPROXIMATELY 5% of the variance in lifetime earnings and lifetime income is explained by variation in initial conditions.

### 6.4 The effect of initial wealth and borrowing constraints

In this section, I show the consequences of a loose borrowing constraint in the capital markets. I replicate the analysis of sections 6.2 and 6.3 with a loose borrowing constraint. First, I show some basic features of the life cycle with loose borrowing constraints.

Figures 14 and 15 show the aggregate income profile and income profiles by education. The fraction of college students is 25%. Since income profile differ less, the Gini coefficient is lower as well. The fraction of agents with zero initial transfer decreases.
Figure 14: Income profile

Figure 15: Income profile by education
Overall, access to credit in this model decreases income inequality over the life cycle.

<table>
<thead>
<tr>
<th>((a, h, \theta))</th>
<th>Lifetime utility</th>
<th>Lifetime earnings</th>
<th>Lifetime wealth</th>
</tr>
</thead>
<tbody>
<tr>
<td>((a_m, \theta_l))</td>
<td>-4.25%</td>
<td>-8.73%</td>
<td>-7.8%</td>
</tr>
<tr>
<td>((a_m, \theta_h))</td>
<td>5.45%</td>
<td>11.1%</td>
<td>9.93%</td>
</tr>
<tr>
<td>((a_l, \theta_m))</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
</tr>
<tr>
<td>((a_h, \theta_m))</td>
<td>21.8%</td>
<td>0.5%</td>
<td>11.8%</td>
</tr>
</tbody>
</table>

Table 7: Lifetime Inequality

<table>
<thead>
<tr>
<th>Y</th>
<th>percentage of variance due to initial conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lifetime income</td>
<td>3.31%</td>
</tr>
<tr>
<td>Lifetime wealth</td>
<td>4.81%</td>
</tr>
<tr>
<td>Lifetime utility</td>
<td>4.05%</td>
</tr>
</tbody>
</table>

Table 8: Variance decomposition

Table 7 shows the changes in lifetime utility, earnings and wealth when there is access to borrowing. The impact of initial wealth does not change much at the bottom of the
distribution, but having it does seem to have an effect on utility at the top. The effects of changes in ability are almost the same as in the case with no borrowing.

Table 8 shows that in an economy with better access to credit, initial conditions matter even less to account for differences in lifetime welfare.

6.5 The effect of parental background and correlation of abilities

The literature on income mobility has shown that income is correlated across generations. In this model, income there is a correlation of income across generations for several reasons: (i) parental transfers allow agents to go to college, which allows parents are children to make similar decisions regarding education, and (ii) correlation of innate abilities across generations.

To study the effect of intergenerational correlation of abilities, I shut down the effect of correlated abilities. Then I simulate the model and perform the same exercise as in sections 6.2 and 6.3.

Figures 14 and 15 show the aggregate income profile and income profiles by education. The fraction of college students is 30%. Income profiles differ a lot. The Gini coefficient increases as the cohort ages. 60% of agents do not receive transfers.

![Income profile graph](image)

Figure 17: Income profile
Figure 18: Income profile by education

Figure 19: Cohort Gini index over the life cycle
<table>
<thead>
<tr>
<th>((a, h, \theta))</th>
<th>Lifetime utility</th>
<th>Lifetime earnings</th>
<th>Lifetime wealth</th>
</tr>
</thead>
<tbody>
<tr>
<td>((a_m, \theta_l))</td>
<td>-4.05%</td>
<td>-7.81%</td>
<td>-7.74%</td>
</tr>
<tr>
<td>((a_m, \theta_h))</td>
<td>5.52%</td>
<td>10.03%</td>
<td>10.2%</td>
</tr>
<tr>
<td>((a_l, \theta_m))</td>
<td>-2.07%</td>
<td>-0.8%</td>
<td>-1.77%</td>
</tr>
<tr>
<td>((a_h, \theta_m))</td>
<td>6.38%</td>
<td>0.49%</td>
<td>4.4%</td>
</tr>
</tbody>
</table>

Table 9: Lifetime Inequality

<table>
<thead>
<tr>
<th>Y</th>
<th>percentage of variance due to initial conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lifetime income</td>
<td>4.43%</td>
</tr>
<tr>
<td>Lifetime wealth</td>
<td>4.81%</td>
</tr>
<tr>
<td>Lifetime utility</td>
<td>4.05%</td>
</tr>
</tbody>
</table>

Table 10: Variance decomposition

Table 9 shows the changes in lifetime utility, earnings and wealth when innate abilities are not correlated across generations.

Table 10 shows that once the channel of intergenerational correlation of abilities is shut down, initial conditions matter slightly less for lifetime welfare. This suggests that parental background is not an important determinant for lifetime welfare.

Results from 9 and 10 suggest that account for intergenerational correlation of abilities is not a particular determinant to study the sources of variation in lifetime welfare.

### 6.6 The increase in college tuition

In the past decades, there has been a sharp increase in college tuition. This section analyzes how the increase in tuition impacts income, wealth and welfare.
Figure 20: Income profile

Figure 21: Income profile by education
Figure 22: Cohort Gini index over the life cycle

<table>
<thead>
<tr>
<th>((a, h, \theta))</th>
<th>Lifetime utility</th>
<th>Lifetime earnings</th>
<th>Lifetime wealth</th>
</tr>
</thead>
<tbody>
<tr>
<td>((a_m, \theta_l))</td>
<td>-4.11%</td>
<td>-7.78%</td>
<td>-7.74%</td>
</tr>
<tr>
<td>((a_m, \theta_h))</td>
<td>5.61%</td>
<td>10.03%</td>
<td>10.2%</td>
</tr>
<tr>
<td>((a_l, \theta_m))</td>
<td>-1.31%</td>
<td>-0.7%</td>
<td>-1.19%</td>
</tr>
<tr>
<td>((a_h, \theta_m))</td>
<td>8.71%</td>
<td>0.76%</td>
<td>6.07%</td>
</tr>
</tbody>
</table>

Table 11: Lifetime Inequality

<table>
<thead>
<tr>
<th>Y</th>
<th>percentage of variance due to initial conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lifetime income</td>
<td>3.9%</td>
</tr>
<tr>
<td>Lifetime wealth</td>
<td>7.24%</td>
</tr>
<tr>
<td>Lifetime utility</td>
<td>2.7%</td>
</tr>
</tbody>
</table>

Table 12: Variance decomposition
7 Conclusions and Final Remarks

Human capital theory is capable to explain several facts about the observed cross-sectional inequality: earnings profiles and Gini coefficients. However, in order to accurately assess the importance of initial conditions in explaining lifetime welfare, several ingredients are missing in this theory. This paper aims to fill that gap, considering a model with intergenerational links and costly human capital accumulation.

Using data from PSID and NLSY I show the evolution of several statistics of interest for a particular cohort: mean earnings, dispersion and skewness. I also show evidence on the correlation of educational attainment across generations, and the importance of wealth and transfers to access to college.

I propose a model with intergenerational links, in terms of abilities and transfers of wealth, in which agents can choose to attend college provided that they have enough resources. Borrowing limits hinder optimal human capital accumulation. Agents would like to accumulate human capital, but they need consumption too. Since they do not have enough wealth, they accumulate less to consume today, which has long-lasting effects. Furthermore, poor agents can’t attend college which also impacts their lifetime earnings. As a result, differences in initial conditions are capable to explain part of the cohort lifetime inequality.

In this paper, the role of financial wealth is through human capital accumulation. Another way in which financial wealth might influence is through inheritances. Although not explicitly modeled in this paper, inherited wealth helps to increase consumption level and smooth its fluctuations against income or human capital shocks. In this regard, inheritances work similarly to transferred wealth not used in human capital accumulation in the model. Several authors have emphasized the role of inheritance in determining observed inequality, such as De Nardi (2004) and Piketty (2014). The role of inheritance is left for future research.

References


G. Becker. *Human Capital: A Theoretical and Empirical Analysis, with Special Reference to*


A More on Empirical Evidence

Figure 23: Earnings age profile by education: NLSY

Figure 23 shows income profiles by education level using data from NLSY79. It depicts similar patterns as Figure 3.
Figures 24 and 25 show the probability of attending college conditional on parents education for two separate subsample, men and women. The overall pattern is that conditioning on both parent being college educated, there is more likely to be college educated. Women series look more volatile.

First, I construct several contingency table showing the proportion of agents going to college conditional on receiving a transfer of at least $500 during those ages. Then I run a
linear probability model and a logit model to explain the probability of going to college conditional on parental transfers.

<table>
<thead>
<tr>
<th>Cohort</th>
<th>No transfer</th>
<th>Transfer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980</td>
<td>0.22</td>
<td>0.56</td>
</tr>
<tr>
<td>1981</td>
<td>0.24</td>
<td>0.52</td>
</tr>
<tr>
<td>1982</td>
<td>0.25</td>
<td>0.50</td>
</tr>
<tr>
<td>1983</td>
<td>0.25</td>
<td>0.48</td>
</tr>
<tr>
<td>1984</td>
<td>0.28</td>
<td>0.44</td>
</tr>
</tbody>
</table>

Table 13: Probability of going to college conditional on transfer

Table 13 shows that the probability of attending college is higher for individuals who received parental transfers during ages 16-22.
<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>log(1 + total_transfer)</td>
<td>0.037***</td>
<td>0.035***</td>
<td>0.026***</td>
<td>0.029***</td>
<td>0.035</td>
<td>0.026</td>
<td>0.029</td>
<td>0.029</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(0.004)</td>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(0.004)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>gender</td>
<td>0.083***</td>
<td>0.104***</td>
<td>0.107***</td>
<td>0.091***</td>
<td>0.109***</td>
<td>0.115***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.020)</td>
<td>(0.021)</td>
<td>(0.025)</td>
<td>(0.021)</td>
<td>(0.021)</td>
<td>(0.026)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>race</td>
<td>0.041***</td>
<td>0.015</td>
<td>0.016</td>
<td>0.043***</td>
<td>0.014</td>
<td>0.015</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.009)</td>
<td>(0.010)</td>
<td>(0.013)</td>
<td>(0.009)</td>
<td>(0.010)</td>
<td>(0.014)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>log(fam_income)</td>
<td>0.077***</td>
<td>0.057**</td>
<td></td>
<td>0.077***</td>
<td>0.063**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.017)</td>
<td>(0.024)</td>
<td></td>
<td>(0.017)</td>
<td>(0.025)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>fam_wealth</td>
<td>0.001***</td>
<td>0.001***</td>
<td></td>
<td>0.001***</td>
<td>0.001***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0001)</td>
<td>(0.0001)</td>
<td></td>
<td>(0.0001)</td>
<td>(0.0001)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>college_res_p</td>
<td>0.175***</td>
<td></td>
<td></td>
<td>0.183***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.034)</td>
<td></td>
<td></td>
<td>(0.034)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>received</td>
<td></td>
<td>0.217***</td>
<td>0.208***</td>
<td>0.144***</td>
<td>0.177***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.021)</td>
<td>(0.021)</td>
<td>(0.022)</td>
<td>(0.026)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.205***</td>
<td>−0.053</td>
<td>−0.314***</td>
<td>−0.316***</td>
<td>0.216***</td>
<td>−0.060</td>
<td>−0.338***</td>
<td>−0.345***</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.045)</td>
<td>(0.070)</td>
<td>(0.105)</td>
<td>(0.013)</td>
<td>(0.045)</td>
<td>(0.070)</td>
<td>(0.106)</td>
</tr>
<tr>
<td>Observations</td>
<td>1,867</td>
<td>1,867</td>
<td>1,635</td>
<td>1,139</td>
<td>1,867</td>
<td>1,867</td>
<td>1,635</td>
<td>1,139</td>
</tr>
</tbody>
</table>

Note: *p<0.1; **p<0.05; ***p<0.01

Table 14: Linear probability model
Table 2 shows the results of a logit model. As in Table 14, all specifications show that the amount of transfers are statistically significant.

B Variance de composition in Dynamic Programming

In a general problem, the present discounted value of utility can be written as

\[ B = \sum_{t=0}^{T} \beta^t u(x_t) \]  

(4)

where \( x_t \) is the state at time \( t \), which may follow a Markov process. The value function is defined as the sum of expected discounted future payoffs, given the initial state \( x \)

\[ J(x) = \mathbb{E}(B|x_0 = x) \]  

(5)

It is useful to define the following operator

\[ M(x) = \mathbb{E}(B^2|x_0 = x) \]  

(6)

Notice that \( J(x) \) and \( M(x) \) can be written as

\[ J(x) = u(x) + \beta P(x'|x)J(x') \]  

(7)

\[ M(x) = u(x)^2 + 2\beta u(x)P(x'|x)J(x') + \beta^2 P(x'|x)M(x') \]  

(8)

The previous two equations can be computed recursively. Using \( J(x) \) and \( M(x) \) we can compute the variance decomposition of a variable

\[ \text{Var}(Y|x) = \mathbb{E}(Y^2|x) - \mathbb{E}(Y|x)^2 = M(x) - J(x)^2 \]  

(9)

To compute the variance decomposition, I use

\[ \text{Var}(Y) = \mathbb{E}(\text{Var}(Y|x)) + \text{Var}(\mathbb{E}(Y|x)) \]  

(10)

The first term on the right hand side are computed using \( M(x) \) and \( J(x) \) and the initial distribution over states \( g(a, h, \theta) \). The second term on the right hand side is computed using the value function and the initial distribution over states.
C Numerical Solution

*Grids* I define grids for ability $\theta$, human capital $h$ and financial capital $a$. The model is solved using parallel computation in the $\theta$ dimension.

*Function approximation* Policy and value functions are solved using collocation methods. I solved the model in the collocation nodes and I interpolate between nodes. I use linear interpolation.

*Optimization problem* The problem is solved using backward induction. At each $t$, the optimization problem is solved using golden search method for one dimensional problem (e.g. during retirement), using a two-dimensional golden method for two-dimensional problems (e.g. during working periods) and a nested golden search for $s$ and a bivariate numerical solver for savings and transfer (to solve for inter-vivos transfers). This last piece is by far the most challenging and computationally expensive part of the algorithm.

*Distributions* Given the discretization of the state space, the policy functions and exogenous shocks define transition matrices over states. The steady state of the model is computed as the ergodic distribution of the transition matrices of the model.