Sources of lifetime inequality revisited

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Abstract

Some factors determined early in life are key determinants of the lifetime value of earnings, consumption and wealth. Furthermore, some of these variables are determined by parental background (ability) or passed on directly from parents (initial wealth). In this paper, I study an overlapping generations economy with a life cycle structure, with borrowing constraints and costly human capital acquisition, in which initial conditions are determined by parental background. The cost of human capital may prevent constrained agents to optimally acquire human capital and intergenerational transmission of wealth may alleviate this effect for wealthy households. With a preliminary calibration, I find the initial wealth is as important as human capital, and ability has a secondary role. This result is in stark contrast with the results previously found in the literature. This result suggests that accounting for borrowing constraints and intergenerational links is key to assess the quantitative impact of initial conditions.

1 Introduction

How important are initial conditions to determine one’s lifetime utility, in terms of consumption and income? Some authors such as Piketty (2014) have argued that financial wealth has become more important to determine one’s lifetime outcomes. Others, such as Hugget, Ventura and Yaron (2011) (HVY, hereafter) argue that financial wealth has little impact to determine one’s lifetime welfare and the observed inequality.

What defines “initial” conditions? In HVY initial conditions refer to financial wealth, human capital and innate ability at age 23, after an important portion of human capital has
been already acquired by formal schooling. Furthermore, in order to accumulate human capital, some financial wealth is necessary upfront, so their estimate of their importance of each source might be biased toward initial human capital. Their framework also lacks of financial constraints that may impede consumption smoothing or even human capital investments.

In this paper, I extend HVY framework to allow for (i) costly human capital, since in order to accumulation human capital, specially at the beginning of life, some financial resources are necessary and (ii) intergenerational transmission of wealth, since parents tend to take care, at least in part, of the cost of their children formal schooling. The addition of these mechanisms, plus borrowing constraint may imply a major contribution of initial wealth in explaining the observed earnings and wealth inequality.

The idea of costly human capital accumulation and intergenerational transmission of wealth goes back to Loury (1981) and Laitner (1992). De Nardi (2004) emphasizes the importance of intergenerational correlation of income in order to better match the wealth and income distribution. Some studies take the intergenerational correlation of income as correlation in productivity/ability between generations. Including financial markets in the process of human capital investments can generate an endogenous intergenerational correlation of earnings, by parent and children choosing the same decision over human capital accumulation. Wealthy families have the resources to provide enough human capital to their offspring.

I study an economy with an overlapping generations structure with altruistic parents. Agents go through three stages in life: education, labor and retirement. During the labor stage, parents must decide how much resources to provide their child to obtain education. Using PSID and NLSY data, I show evidence about how mean earnings and inequality measures evolve in the life cycle for a cohort. I also show evidence of the persistence of educational choices across generations and the importance of wealth in obtaining education, measured as owned wealth and parental transfer.

To highlight the importance of the capital markets, I study a simple life cycle framework without intergenerational transfers under two extreme cases of capital markets: complete markets and no capital markets. The process of human capital is affected by the lack of capital markets. This simple model is capable of replicating the properties of inequality of a cohort in the life cycle, as well as the mean income profile. Allowing for different occupations and agents with different access to capital markets increases the earnings
inequality generated by the model. In a simple quantitative exercise, I conclude that having access to better financial conditions can have great impact in the present discounted value of consumption, utility and income.

The rest of the paper is organized as follows. Section 2 reviews the literature. Section 3 describe the life cycle model. Section 4 uses PSID and NLSY data to show the evolution over the life cycle of mean earnings and several measures of inequality for a particular cohort, as well as the importance of parental education, financial wealth and transfer in the process of human capital investment. Section 5 discuss a simplified model to highlight the importance of capital markets in the process of human capital accumulation. Section 6 discuses the main findings from the model and Section 7 concludes.

2 Related Literature

The idea of intergenerational transmission of wealth from parents to children and human capital accumulation dates back to Loury (1981). In his model, parents are subject to borrowing constraints and therefore they may not be able to optimally invest in their child’s human capital. This mechanism endogeneously generates the earnings distribution in the economy. Despite that this mechanism has been well-known for decades, the quantitative impact of this mechanism has not being explored in quantitative macroeconomic models.

There are several papers studying intergenerational transmission of wealth, De Nardi (2004), Boar (2017), Luo (2017), to name a few. Most of them study bequests and/or intervivos transfers, but none of them study the relationship between transfers and human capital accumulation.

There is a vast literature explaining earnings inequality during life cycle, for instance Storesletten, Telmer and Yaron (2001), Hugget, Ventura and Yaron (2006). See Meghir and Pistaferri (2011) for a survey.

Daurich (2017) studies an economy with intervivos transfers and human capital accumulation, where parents influence their child’s human capital by investing time and money. His focus is different in that he studies the macroeconomic implication of changing schooling financing by changing the tax policy, whereas my focus is to quantitatively disentangle the effects of initial conditions, namely learning ability, initial human capital and initial wealth, in order to explain the lifetime inequality.
Credit constraints and human capital acquisition have been study in Lochner and Monje-Naranjo (2011) and Hai and Heckman (2017). They focus in the interaction of credit constraints human capital rather than their implications for inequality and welfare.

Abbott et. al. (2018) study an economy where financial frictions play a role in human capital acquisition. Their main focus is the implication of education policy in determining intervivos transfers.

The relative importance of shocks versus initial condition in order to explain lifetime welfare has been study in Huggety, Ventura and Yaron (2006) and Huggett, Ventura and Yaron (2011). My paper contributes to this literature by re assessing the relative contribution of each factor in an economy with intergenerational links across generations and costly human capital accumulation.

3 Economic Environment

Demographics: Life cycle model with overlapping generation structure and deterministic finite lives. At age 30 agents have a child. Agents die for sure at age 80.

Heterogeneity: Agents are heterogeneous in innate ability $\theta$, human capital $h$ and financial assets $a$. Innate ability is determined at the time of birth and is fixed throughout agent’s lifetime. In the baseline model, I assume ability is and i.i.d shock across generations, but correlated with initial human capital\(^1\). Later I will allow for intergenerational correlation of abilities. Ability allows agents to acquire human capital faster, so it can be interpreted as learning ability. Agents have two sources of human capital accumulation (i) acquire formal education (going to college) (ii) acquire human capital spending time during their working life (e.g. on the job training), à la Ben-Porath. During working stage agents receive i.i.d. idiosyncratic shocks to human capital, $z_t$. Financial assets are accumulated through savings during lifetime.

Life cycle: Agents go through three stages in their lives: (i) education, (ii) labor and (iii) retirement. In the first stage of their lives agents decide whether to go to college. This

\(^1\)Huggett et al. (2011) argue that to replicate the increasing inequality over the life cycle a positive correlation between ability and initial human capital is necessary. Guvenen et al. (2013) offer a rationale for this conditions, since agents enter the model at some point in their lifetime, they have already accumulate some human capital in early stages in life. Thus, agents with higher ability accumulate more human capital which generate the positive correlation.
decision depends on their innate ability, initial human capital as well as the availability of financial resources. If they choose to go to college, agents acquire high skill human capital, otherwise they stay as low skilled workers. Being high skilled worker can be interpreted as working in certain “occupations”, “industries” or “sectors” that pay higher wage rates per unit of human capital.

During their labor stage, agent decide how much time spend in labor activities and how much time to spend acquiring human capital. Human capital is affected by individual investments as well as stochastic shocks. By age 30, individual’s child is born. By age 48 the child becomes and adult and his parent must decide how much financial resources to transfer him\(^2\).

During retirement agents receive social security benefits and no labor earnings. There are no bequests, so agents consume all their wealth at the end of their lives. Agents die for sure at age 80.

**Intergenerational Links:** In the baseline model I assume no links between neither ability nor initial human capital across generations. Later I study the effects of intergenerational correlation of ability. Wealth is transmitted across generation only through the inter vivos transfer, no bequests.

**Borrowing constraints:** In order to fully assess the contribution of borrowing constraint to human capital accumulation I use different borrowing limits: (i) no borrowing allowed, (ii) ad-hoc borrowing limits.

**Preferences:** Let \( U_k \) be the lifetime utility derived from consumption flow for agent of generation \( k \), \( U_k = \sum_{t=1}^{T} \beta^t u(c_t) \), where \( u(c_t) \) is increasing and concave function. Individuals maximize the lifetime utility from their own consumption and weight their child’s lifetime utility of consumption. They maximize \( U_k = U_k + \tilde{\gamma} U_{k+1} \), where \( \tilde{\gamma} \) is the weight parents put on their kid’s utility.

**Tax schedule and Government spending:** The tax and transfer system has two components: an income tax component and a social security system component \( T = T^{ss} + T^{inc} \). The social security part consist in a proportional tax \( \tau^{ss} \) for active workers, and a positive transfer equal to a fraction of average earnings in the economy, as in Huggett et al. (2011). The income tax schedule captures the effective average income tax rates of the economy

\(^2\)It is assumed that once the wealth transfer is made, the child is free to spend those resources in education or consumption. Parents do not have a the technology to monitor how kids spends resources nor have the ability to make contingent transfers.
as function of income. In particular, tax rate function $t(\tilde{y}) = 1 - \mu \tilde{y}^{-\tau}$, as in Bernabou (2002) and Heathcote et al. (2014), where $\tilde{y}$ is taxable income. I consider taxable income as the after social security tax labor income plus capital income. Guner et al. (2014) show that this tax function specification provides a reasonable fit to the actual U.S. income tax schedule for both the average tax rates and effective marginal tax rates. The government has a balanced budget every period.

### 3.1 The Life cycle model

**Stage 1: Education stage**

Agents choose whether to go to college or not. The value of an agent at time zero, with initial financial wealth $a$, human capital $h$ and ability $\theta$ is:

$$V_0(a, h; \theta) = \max \{V^c(a, h; \theta), V^{nc}(a, h; \theta)\}$$

where $V^c(k, h; a)$ is the value of a college student and $V^{nc}$ is the value of not going to college.

$$V_j(a_j, h_j; \theta) = \max_{c_j, s_j} u(c_j) + \beta \mathbb{E}_z[V_{j+1}(a_{j+1}, h_{j+1}; \theta)]$$

s.t. $c_j + a_{j+1} = a_j (1 + r) - P_c$

If the agent chooses not to go to college, she starts working as low skilled worker.

**Stage 2: Working stage**

Agents work either as a high skilled worker or low skilled worker, depending on whether they go to college. High skilled workers receive a wage $w_h$ per unit of time and human capital, whereas low skilled workers receive $w_l$.

The value function of a worker hired at wage rate $w$ is

$$V_j(a_j, h_j; \theta) = \max_{c_j, s_j} u(c_j) + \beta \mathbb{E}_z[V_{j+1}(a_{j+1}, h_{j+1}; \theta)]$$

s.t. $c_j + a_{j+1} = a_j + \mu (ra_j + (1 - \tau_{ss}) wh_j (1 - s_j))^{1-\tau}$

$$h_{j+1} = \exp(z_{j+1})((1 - \delta) h_j + \theta (s_j h_j)^a)$$
where \( w \) equals \( w_l \) or \( w_h \).

At age 48, when the individual’s child becomes an adult, the parent must decide how much wealth to transfer. At this age, the value function is

\[
V_j(a_j, h_j; \theta) = \max_{c_j, s_j} u(c_j) + \beta \mathbb{E}[V_{j+1}(a_{j+1}, h_{j+1}; \theta)] + \gamma V^o(a_0^*)
\]

s.t. \( c_j + a_{j+1} + a_0^* = a_j + \mu(ra_j + (1 - \tau ss)wh_j(1 - s_j))^{1-\tau} \)

\[ h_{j+1} = \exp(z_{j+1})((1 - \delta)h_j + \theta(s_jh_j)^{\alpha}) \]

where \( V^o(a_0^*) \) is the expected value of lifetime utility of the son with initial wealth \( a_0^* \),

\[
V^o(a_0^*) = \mathbb{E}_{h,\theta}(V_0(a_0^*, h, \theta))
\]

### Stage 3: Retirement

Retirees live off their wealth and receive a constant pension benefit \( p \). The retiree’s problem is

\[
V(a_j) = \max_{c_j} u(c_j) + \beta V(a_{j+1})
\]

s.t. \( c_j + a_{j+1} = p + (1 + r)a_j \)

### 3.2 Equilibrium

A Steady State Competitive Equilibrium is a set of policy functions \( \{c_t, s_t\}_{t=0}^T \), value functions \( \{V_t(a, h; \theta)\}_{t=0}^T \) and distribution over states \( \{g_t(h, \theta)\}_{t=0}^T \) such that given factor prices \( r, w_l, w_h \), government spending and tax schedule \( \{T, G\} \) and initial distribution over states \( g_0(h, \theta) \):

1. Given prices and government policies, policy functions and value functions solve individual’s problem.

2. Distributions are consistent with policy functions and initial distribution.
4 Empirical Analysis

I start this section by describing the data sources and the sample selection criteria. Then I document several facts. First, I document mean income profiles and inequality measures by cohort, in a similar fashion as in HYY. Next, I document income profiles by education. Then I document the persistence of education levels across generations. Afterwards, I document education levels by wealth class. Finally, I document the probability of attending to college conditional on parental transfers.

4.1 Data sources

The Panel Study of Income Dynamics started in 1968 and surveys more than 18,000 individuals from 5,000 families and over five generations, representative of the United States. Annual waves up to 1997 and biannually since then. It collects data on family and individuals, with more details on household head and wife. After a child leaves home and form his own household, his family is also part of the survey. In this paper I use mainly information on household head.

The National Longitudinal Survey of Youth (NLSY) surveys individuals aged 14 to 22 at the time of their first interview. There are two different cohort, NLSY79 which is first interviewed in 1979 and NLSY97 which interviews youths for the first time in 1997. After the first interview they follow the cohort with annual or biannual surveys. NLSY79 and NLSY97 have a sample size of around 12,000 and 9,000 individuals approximately, respectively. I use information from these cohorts to compute information of earnings as well as parental transfer during young ages.

4.2 Income profiles in the PSID

To compute income profiles I focus on male household heads from the SCR sample (nationally representative). For individuals aged less or equal than 30, they have to work more than 260 hours a year and earn at least $1,000 a year (1970 dollars). Individuals older than 30 are kept in the sample if they earn at least $1,500 and work more than 520 hours. To construct statistics, I consider a 5 year bin for each age.

Ideally, I’d like to estimate age profile for several statistics by running the following regression
\[ \text{stat}_{i,j} = \text{age effect}_i + \text{time effect}_j + \text{cohort effect}_c + \varepsilon_{i,j} \] (1)

where \( \text{stat}_i \) is the statistics of interest for age \( i \).

To estimate this equation I can control for either time fixed effect or cohort fixed effect. Controlling for both is not possible, since age, time and year are collinear. I show results by controlling for one of these effects at a time. For a discussion on the differences between controlling for time or cohort effects see Heathcote et al. (2005).

Panel A in Figure 1 shows the mean earnings age profile. The differences between cohort and time effect are not noticeable until the end of life cycle. The figure displays an increase and concave pattern, with a small decreasing pattern at the end when controlling for time effect.

Panel B in Figure 1 displays the evolution of cohort earnings inequality as the cohort ages. The pattern is increasing. Unlike the earnings profile, the shape of the Gini coefficient profile depends crucially on whether we control for time or cohort effect.
4.3 Incomes profiles by education

Now I focus on just mean earnings, and compute the same statistical model as (1) with two different subsamples: one with college educated individuals and one with level of education less or equal than high school graduate. I use the same criteria as the previous section to select the sample.

Figure 2 shows the income profile for college educated and non-college educated workers using PSID data. In appendix, I show that a similar earnings age profile is observed in the NLSY data.

![Earnings age profile by education: PSID](image)

Earnings profiles differ by education attainment. Earnings profile for college educated worker is steeper and achieves higher level at the end of the life cycle than that of high school educated workers. Several authors have documented heterogeneous income profiles, for instance Guvenen (2009). Figure 2 is consistent with that view.

4.4 Intergenerational persistence of education

I compute several statistics of the persistence of education across generations. I consider waves from 1974-2015 because there is enough information about mother’s and father’s education. I focus on the group of 23-27 years old (25 years old bin).
After creating dummies of college education for individuals, mothers and fathers, I compute contingency tables for education, so I can compute conditional probabilities of being college educated, conditional on mothers’, fathers’, and both parent being college educated. Analyses by gender are shown in the appendix.

Figure 3: Probability of being college educated, conditional on parent’s education

Figure 3 shows the probability of going to college conditional on parent’s education. Panel A shows that conditional on both parent being college educated, the probability of being college educated is higher than the probability when only one parent went to college, for all years. Panel B shows conditional on both parent being non-college educated, the probability of being college educated is also higher, although quantitatively small, than the probability when conditioning in only one parent being non-college educated. Both graphs display an upward trend, which represent increase in enrollment in college education. There are not quantitatively important differences when doing the analysis by gender (shown in the appendix).

4.5 Education by wealth class

In this section I explore the conditioning probability of attending college, given household wealth. Again, I focus on individuals aged 23-27. I constructed a measure of wealth class, according to the wealth distribution among those individuals. Each wealth class has the same size. Wealth class 1, 2 and 3 correspond to the bottom, medium and upper side of the

![Education and wealth](image)

Figure 4: Probability of being college educated, conditional on household wealth

Figure 4 shows probability of being college educated conditional on wealth class. The probability of being college educated conditional on being in the upper wealth class is higher for every year available. In recent years, the probability of attending college condition on being in the wealthy class has skyrocketed, from 23% in 2005 (its lowest point in the whole period) up to roughly 50% in recent years, which suggest that wealth is more important to access to higher education in recent years. The conditional probability of going to college does not differ much for the bottom two wealth class. In appendix I show similar evidence with a definition of wealth that excludes housing. Results are robust to the definition of wealth.

This analysis focuses on household wealth, and not on transferred wealth. At young ages, however, it is most likely that a great portion of wealth corresponds to transferred wealth. To better assess the contribution of transfer I focus on whether the household has
received gifts in recent years and its impact on education.

4.6 Education and parental transfers: Evidence from NLSY97


I focus on cohorts 1980 and 1981 because the information about transfers is more accurate and consistent across years\(^3\). I construct a variable with the sum of parental transfers to the child, excluding allowances, from ages 16 to 22.

<table>
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<tr>
<th>Cohort</th>
<th>Q1</th>
<th>Median</th>
<th>Mean</th>
<th>Q3</th>
<th>Fraction Tr &gt;0</th>
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<td>1980</td>
<td>300</td>
<td>750</td>
<td>3493</td>
<td>2200</td>
<td>39.4%</td>
</tr>
<tr>
<td>1981</td>
<td>300</td>
<td>850</td>
<td>3292</td>
<td>2138</td>
<td>39.9%</td>
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</table>

Table 1: Descriptive statistics transfers

Table 1 shows some basic descriptive statistics about total transfers for cohorts 1980 and 1981. The histograms do not differ much. The distribution of transfer is extremely skewed, almost 40% receive no transfer, and the mean is more than three times the median. The maximum total transfer for each year is about 80,000.

\(^3\)During the first years of the survey, respondents give information about transfers received from mother and father, separately. If they do not provide an exact number, they can give an estimate. This information is enough to construct a reasonable estimate of parental transfers during ages 16-22. For later cohorts, the survey changes a bit, and it is impossible to identify the sources of transfer (they asked from transfers from other relatives as well). The exact amount of transfer is also lost, and there are only estimates of transfer. Transfers for Cohorts 1980 and 1981 rely less on those estimates.
Figure 5 shows the histogram for positive transfers for both years. I trim the scale to display the histogram appropriately. The histogram displays a high positive skewness.

I estimate several specifications of a logit model to assess the importance of transfers in the likelihood of attending college. Since some amount of the total transfer is imputed, I also consider a dummy variable of whether the respondent received a transfer.

Table 2 summarizes the results of a logit probability model with the a dummy variable of being college-educated as independent variable. To control for transfers, I use both a continuous variable as well as a dummy variable. I also control for demographic characteristics, income and wealth and parental education. In all specification total transfers is statistically significant with 1 percent of confidence. In appendix, I show that I obtain similar results when I estimate a linear probability model.

To interpret the coefficients, I compute the Average Marginal Effect (AME) and the Marginal Effect at Means (MEM) from all specifications with the dummy variable of transfers.
### Table 2: Logit model

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<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
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<td>0.0001***</td>
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<td>0.0001***</td>
<td>0.0001***</td>
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<td>received</td>
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<td>-2.791***</td>
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<td></td>
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<td>(0.073)</td>
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<td>(0.255)</td>
<td>(0.458)</td>
<td>(0.461)</td>
<td>(0.601)</td>
<td>(0.614)</td>
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*Note:* *p<0.1; **p<0.05; ***p<0.01

### Table 3: Marginal Effects

<table>
<thead>
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<th>(2)</th>
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<th>(8)</th>
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<td>0.207</td>
<td>0.140</td>
<td>0.1762</td>
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<td>MEM</td>
<td>0.217</td>
<td>0.210</td>
<td>0.154</td>
<td>0.2015</td>
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</table>

Table 3: Marginal Effects
Table 3 shows that in every specification, the average marginal effect and the marginal
effect at the mean go from 15% up to 20%. This means that, on average, there is a 20%
extra chance of going to college for someone who has transfers, everything else constant.
This is roughly in line with the estimates shown in the linear probability model discussed
in the appendix.

## 5 Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Meaning</th>
<th>Value</th>
<th>Source/Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r )</td>
<td>rate of return</td>
<td>4%</td>
<td>calibrated</td>
</tr>
<tr>
<td>( w_l )</td>
<td>low wage</td>
<td>1</td>
<td>normalization</td>
</tr>
<tr>
<td>( w_h )</td>
<td>high wage</td>
<td>1.6</td>
<td>(Goldin and Katz (2007))</td>
</tr>
<tr>
<td>( p )</td>
<td>pension</td>
<td>55</td>
<td>40% of mean earnings</td>
</tr>
<tr>
<td>( P_c )</td>
<td>price of college</td>
<td>129</td>
<td>share college educated agents (30%)</td>
</tr>
</tbody>
</table>

### Preferences and human capital

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Meaning</th>
<th>Value</th>
<th>Source/Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>Discount Factor</td>
<td>0.95</td>
<td>Standard</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>CRRA utility function</td>
<td>2</td>
<td>Standard</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>production function</td>
<td>0.7</td>
<td>HVY (2011)</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Parameter of Altruism</td>
<td>0.2446</td>
<td>(calibrated)</td>
</tr>
<tr>
<td>( \delta )</td>
<td>Human capital depreciation</td>
<td>0.007</td>
<td>(calibrated)</td>
</tr>
<tr>
<td>( \sigma_z )</td>
<td>Variance human capital shocks</td>
<td>0.05</td>
<td>(calibrated)</td>
</tr>
</tbody>
</table>

### Government

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Meaning</th>
<th>Value</th>
<th>Source/Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu )</td>
<td>Tax constant</td>
<td>0.902</td>
<td>Guner et. al (2014)</td>
</tr>
<tr>
<td>( \tau )</td>
<td>Tax progressivity</td>
<td>0.036</td>
<td>Guner et. al (2014)</td>
</tr>
<tr>
<td>( \tau_{ss} )</td>
<td>Social security tax</td>
<td>0.106</td>
<td>HVT(2011)</td>
</tr>
</tbody>
</table>

### Distribution

\[(\mu_h, \mu_\theta, \sigma_h, \sigma_\theta, \rho_{h, \theta})\] initial distribution (log normal) (90, 0.017, 56, 0.06, 0.43) calibrated

Table 4: Baseline Calibration

Table 4 reports the parameters used in the solution of the simple models. The discount
factor \( \beta \) is set to 0.95, a common value used in the literature for an annual discount factor.
The risk aversion parameter \( \sigma \) is set to 2. The interest rate is set to 4\%. The wage for the low wage sector is normalized to 1. The wage in the high wage sector is set to 1.6, that is, there is a 60\% skill premium, as documented by Goldin and Katz (2007). The concavity of the human capital production function is set to 0.7, in line with estimates shown in Huggett et al. (2011) and Guvenen et al. (2013). The pension benefit and the price of college are calibrated to target a 40\% replacement rate and a 30\% of college educated agents. The tax
schedule is taken from Guner et al. (2014) who show that the tax function in this paper is a good approximation of the US tax scheme. The social security tax rate is 10.6%, as in Huggett et al. (2011). The parameters of the initial distribution as well as the parameter for altruism and human capital shocks are calibrated to match properties of the evolution of cohort inequality over the life cycle. The calibration is still preliminary.

6 Analysis

6.1 Model Fit

Figure 6 shows the income profile from the baseline model. It captures the profile observed in the data.

![Figure 6: Income profile](image)
Figure 7 shows the income profile by education groups. It shows a similar pattern as 2.

Figure 8 shows the evolution of the Gini coefficient over the life cycle. It displays a small increase over the life cycle, as suggested by the time effect Gini estimates shown in 1. The level is lower than the data.
Figure 9: Initial wealth - parental transfers

Figure 9 shows the initial wealth distribution generated by the model. It is highly skewed consistent with the empirical evidence. 51% of agents receive positive transfers. This is close to the empirical observation of 40%. The fraction of college-educated agents in the model is 45%.

6.2 Sources of Lifetime Inequality

In this section I compare the present discounted value of income, wealth and utility by changing 1) the level initial wealth, 2) the level of initial human capital and 3) innate ability. Table 5 summarizes the findings. I consider changes with respect to the median. For human capital, I consider the 25th and 75th percentile ($h_l, h_h$). For ability, I consider the 30th and 70th percentile ($\theta_l, \theta_h$). For initial wealth I consider the 75th percentile $a_h$. Values expressed as variations with respect to the respective value evaluated at the medians $(a_m, h_m, \theta_m)$.

---

Utility changes are expressed as consumption equivalent variations.
Table 5 shows the changes in lifetime utility, earnings and wealth when there is a change in initial conditions. Changes in wealth and in initial human capital have a great impact in lifetime welfare. The effect of ability is lower. These results are in stark contrast with Huggett et al. (2011), who argue that among initial conditions, ability and human capital have a great impact and that of initial wealth was almost negligible. There are two reasons that explain the different findings. First, they do not take into account that financial wealth is necessary to acquire specific human capital. For some agents, having a small extra human capital has a big impact since they are now able to go to college. Second, they do not consider borrowing constraints. In the baseline model, agents are not allowed to borrow to get education. In the next section, I explore the effect of having different borrowing constraints in lifetime welfare.

6.3 Variance decomposition

In this section I want to study the variance decomposition of lifetime income, lifetime wealth and lifetime utility. I want to decompose the variance in two: variance associated to differences in initial condition vs variance associated to differences in shocks received during lifetime.

For a variable $Y$ (lifetime income, lifetime wealth, lifetime utility), I can use the following decomposition

\[
\text{var}(Y) = \text{E}(\text{var}(Y|X)) + \text{var}(\text{E}(Y|X))
\]

(2)

where $X$ is a vector of initial conditions. In appendix I show how to compute each of

<table>
<thead>
<tr>
<th>$(a, h, \theta)$</th>
<th>Lifetime utility</th>
<th>Lifetime earnings</th>
<th>Lifetime wealth</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(a_m, h_t, \theta_m)$</td>
<td>-31.1%</td>
<td>-34.5%</td>
<td>-34.6%</td>
</tr>
<tr>
<td>$(a_m, h_h, \theta_m)$</td>
<td>42.8%</td>
<td>49.2%</td>
<td>49.3%</td>
</tr>
<tr>
<td>$(a_m, h_m, \theta_t)$</td>
<td>-1.51%</td>
<td>-3.19%</td>
<td>-3.2%</td>
</tr>
<tr>
<td>$(a_m, h_m, \theta_h)$</td>
<td>1.56%</td>
<td>3.19%</td>
<td>3.2%</td>
</tr>
<tr>
<td>$(a_h, h_m, \theta_m)$</td>
<td>30.5%</td>
<td>32.6%</td>
<td>42.4%</td>
</tr>
</tbody>
</table>

Table 5: Lifetime Inequality
these terms in a dynamic programming problem.

<table>
<thead>
<tr>
<th>Y</th>
<th>percentage of variance due to initial conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lifetime income</td>
<td>23.5%</td>
</tr>
<tr>
<td>Lifetime wealth</td>
<td>23.3%</td>
</tr>
<tr>
<td>Lifetime utility</td>
<td>11.7%</td>
</tr>
</tbody>
</table>

Table 6: Variance decomposition

Table 6 shows the fraction of variance in lifetime utility, lifetime earnings and lifetime wealth associated to initial conditions. More than 20% of the variance in lifetime earnings and lifetime income is explained by variation in initial conditions.

6.4 The effect of initial wealth and borrowing constraints

In this section I show the consequences of a loose borrowing constraint in the capital markets. I replicate the analysis of sections 6.2 and 6.3 with a loose borrowing constraint. First, I show some basic features of the life cycle with loose borrowing constraints.

Figures 10 and 11 show the aggregate income profile and income profiles by education. The fraction of college students is 65%. Since income profile differ less, the Gini coefficient is lower as well. The fraction of agents with zero initial transfer is zero.
Figure 10: Income profile

Figure 11: Income profile by education
Overall, access to credit in this model decreases income inequality over the life cycle.

<table>
<thead>
<tr>
<th>( (a, h, \theta) )</th>
<th>Lifetime utility</th>
<th>Lifetime earnings</th>
<th>Lifetime wealth</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (a_m, h_t, \theta_m) )</td>
<td>-29.5%</td>
<td>-34.3%</td>
<td>-33.3%</td>
</tr>
<tr>
<td>( (a_m, h_h, \theta_m) )</td>
<td>41.9%</td>
<td>48.9%</td>
<td>47.5%</td>
</tr>
<tr>
<td>( (a_m, h_m, \theta_l) )</td>
<td>-2.16%</td>
<td>-4.4%</td>
<td>-4.27%</td>
</tr>
<tr>
<td>( (a_m, h_m, \theta_h) )</td>
<td>2.26%</td>
<td>4.4%</td>
<td>4.27%</td>
</tr>
<tr>
<td>( (a_h, h_m, \theta_m) )</td>
<td>20.5%</td>
<td>23.8%</td>
<td>25.7%</td>
</tr>
</tbody>
</table>

Table 7: Lifetime Inequality

<table>
<thead>
<tr>
<th>( Y )</th>
<th>percentage of variance due to initial conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lifetime income</td>
<td>22.9%</td>
</tr>
<tr>
<td>Lifetime wealth</td>
<td>22.5%</td>
</tr>
<tr>
<td>Lifetime utility</td>
<td>9.35%</td>
</tr>
</tbody>
</table>

Table 8: Variance decomposition

Table 7 shows the changes in lifetime utility, earnings and wealth when there is access to borrowing. Clearly, the impact of initial wealth decreases substantially... The effect of
changes in initial human capital and ability are almost the same as in the case with no capital markets.

Table 8 shows that in an economy with better access to credit, initial conditions matter less to account for differences in lifetime welfare.

6.5 The effect of parental background and correlation of abilities

There are several pieces of evidence that suggest that human capital and abilities are correlated across generations. The evidence is based on income correlation across generations. In this model, income is correlated across generations for several reasons: (i) parental transfers allow agents to go to college, and these children will transfer themselves wealth to their offspring, (ii) correlation of initial human capital and (iii) correlation of innate abilities across generations.

To study the effect of intergenerational correlation of human capital, I assume that innate abilities are perfectly correlated across generations. Then I simulate the model and perform the same exercise as in sections 6.2 and 6.3.

Figures 10 and 11 show the aggregate income profile and income profiles by education. The fraction of college students is 44.8%. Income profiles differ a lot. The Gini coefficient decreases at the beginning of the life cycle and then it increases. 51.1% of agents do not receive transfers.

5Since the intergenerational correlation of abilities equals one then the model displays several ergodic distributions. In order to obtain a unique ergodic distribution, I initialize the model with a distribution over abilities equal to the marginal distribution of abilities of the joint distribution of human capital and abilities of the baseline model
Figure 13: Income profile

Figure 14: Income profile by education
Table 9: Lifetime Inequality

<table>
<thead>
<tr>
<th>$(a, h, \theta)$</th>
<th>Lifetime utility</th>
<th>Lifetime earnings</th>
<th>Lifetime wealth</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(a_m, h_l, \theta_m)$</td>
<td>-31.3%</td>
<td>-34.5%</td>
<td>-34.8%</td>
</tr>
<tr>
<td>$(a_m, h_h, \theta_m)$</td>
<td>43.1%</td>
<td>49.2%</td>
<td>49.5%</td>
</tr>
<tr>
<td>$(a_m, h_m, \theta_l)$</td>
<td>-1.45%</td>
<td>-3.09%</td>
<td>-3.11%</td>
</tr>
<tr>
<td>$(a_m, h_m, \theta_h)$</td>
<td>1.49%</td>
<td>3.09%</td>
<td>3.11%</td>
</tr>
<tr>
<td>$(a_h, h_m, \theta_m)$</td>
<td>32.7%</td>
<td>34.5%</td>
<td>44.6%</td>
</tr>
</tbody>
</table>

Table 10: Variance decomposition

<table>
<thead>
<tr>
<th>Y</th>
<th>percentage of variance due to initial conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lifetime income</td>
<td>25.6%</td>
</tr>
<tr>
<td>Lifetime wealth</td>
<td>26.2%</td>
</tr>
<tr>
<td>Lifetime utility</td>
<td>12.5%</td>
</tr>
</tbody>
</table>

Table 9 shows the changes in lifetime utility, earnings and wealth when innate abilities are perfectly correlated across generations. The impact of initial wealth, initial human capital and ability are in line with the baseline case.
Table 10 shows that once accounted for intergenerational correlation of abilities, initial conditions matter slightly more for lifetime welfare. In the baseline model, the initial distribution of abilities and human capital was i.i.d. across generations. This time, considering the extreme case when abilities are perfectly correlated across generations, the fraction of the variation in welfare explained by initial conditions is almost the same.

Results from 9 and 10 suggest that account for intergenerational correlation of abilities is not a particular determinant to study the sources of variation in lifetime welfare.

7 Conclusions and Final Remarks

Human capital theory is capable to explain several facts about the observed cross-sectional inequality: earnings profile, dispersion and skewness measures. However, in order to accurately assess the importance of initial conditions in explaining lifetime welfare, several ingredients are missing in this theory. This paper aims to fill that gap, considering a model with intergenerational links and costly human capital accumulation.

Using data from PSID and NLSY I show the evolution of several statistics of interest for a particular cohort: mean earnings, dispersion and skewness. I also show evidence on the importance of parental education and wealth to achieve higher education.

In a simplified model, I show that different arrangement of the capital markets can have an important impact on the process of human capital accumulation and thus in lifetime earnings. This highlights the importance of wealth in determining lifetime utility. A simple calibrated version of the model replicates several facts of the evolution of mean earnings and inequality measure over the life cycle of a cohort. The model also suggests that the contribution of financial wealth is higher than what the literature has found previously.

In this paper the role of financial wealth is through human capital accumulation. Other ways in which financial wealth might influence is through inheritances. Although not explicitly modeled in this paper, inherited wealth helps to increase consumption level and smooth its fluctuations against income or human capital shocks. In this regard, inheritances work similar as transferred wealth not used in human capital accumulation in the model. Several authors have emphasized the role of inheritance in determining observed inequality, such as De Nardi (2004) and Piketty (2014). The role of inheritance is left for future research.
References


Figure 16: Earnings age profile by education: NLSY

Figure 16 shows income profiles by education level using data from NLSY79. It depicts similar patterns as Figure 2.
Figures 17 and 18 show the probability of attending college conditional on parents’ education for two separate subsample, men and women. The overall pattern is that conditioning on both parent being college educated, there is more likely to be college educated. Women series look more volatile.
Figure 19: Probability of being college educated, conditional on household wealth (no housing)

Figure 19 displays similar pattern as Figure 4. It shows that the inclusion of housing in the definition of wealth does not make much difference to explain college attendance.

First, I construct several contingency table showing the proportion of agents going to college conditional on receiving a transfer of at least $500 during those ages. Then I run a linear probability model and a logit model to explain the probability of going to college conditional on parental transfers.
Table 11: Probability of going to college conditional on transfer

<table>
<thead>
<tr>
<th>Cohort</th>
<th>No transfer</th>
<th>Transfer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980</td>
<td>0.22</td>
<td>0.56</td>
</tr>
<tr>
<td>1981</td>
<td>0.24</td>
<td>0.52</td>
</tr>
<tr>
<td>1982</td>
<td>0.25</td>
<td>0.50</td>
</tr>
<tr>
<td>1983</td>
<td>0.25</td>
<td>0.48</td>
</tr>
<tr>
<td>1984</td>
<td>0.28</td>
<td>0.44</td>
</tr>
</tbody>
</table>

Table 11 shows that the probability of attending college is higher for individuals who received parental transfers during ages 16-22.
<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent variable:</td>
<td>educ</td>
<td>educ</td>
<td>educ</td>
<td>educ</td>
<td>educ</td>
<td>educ</td>
<td>educ</td>
<td>educ</td>
</tr>
<tr>
<td>total_transfer</td>
<td>0.00001***</td>
<td>0.00001***</td>
<td>0.00001***</td>
<td>0.00001***</td>
<td>0.00001***</td>
<td>0.00001***</td>
<td>0.00001***</td>
<td>0.00001***</td>
</tr>
<tr>
<td></td>
<td>(0.00000)</td>
<td>(0.00000)</td>
<td>(0.00000)</td>
<td>(0.00000)</td>
<td>(0.00000)</td>
<td>(0.00000)</td>
<td>(0.00000)</td>
<td>(0.00000)</td>
</tr>
<tr>
<td>received</td>
<td>0.217***</td>
<td>0.208***</td>
<td>0.144***</td>
<td>0.177***</td>
<td>0.177***</td>
<td>0.177***</td>
<td>0.177***</td>
<td>0.177***</td>
</tr>
<tr>
<td></td>
<td>(0.021)</td>
<td>(0.021)</td>
<td>(0.022)</td>
<td>(0.026)</td>
<td>(0.026)</td>
<td>(0.026)</td>
<td>(0.026)</td>
<td>(0.026)</td>
</tr>
<tr>
<td>gender</td>
<td>0.083***</td>
<td>0.091***</td>
<td>0.104***</td>
<td>0.109***</td>
<td>0.111***</td>
<td>0.115***</td>
<td>0.115***</td>
<td>0.115***</td>
</tr>
<tr>
<td></td>
<td>(0.021)</td>
<td>(0.021)</td>
<td>(0.021)</td>
<td>(0.026)</td>
<td>(0.026)</td>
<td>(0.026)</td>
<td>(0.026)</td>
<td>(0.026)</td>
</tr>
<tr>
<td>race</td>
<td>0.047***</td>
<td>0.043***</td>
<td>0.014</td>
<td>0.014</td>
<td>0.014</td>
<td>0.014</td>
<td>0.014</td>
<td>0.014</td>
</tr>
<tr>
<td></td>
<td>(0.009)</td>
<td>(0.009)</td>
<td>(0.010)</td>
<td>(0.010)</td>
<td>(0.010)</td>
<td>(0.010)</td>
<td>(0.010)</td>
<td>(0.010)</td>
</tr>
<tr>
<td>log(fam_income)</td>
<td>0.091***</td>
<td>0.077***</td>
<td>0.069***</td>
<td>0.063***</td>
<td>0.063***</td>
<td>0.063***</td>
<td>0.063***</td>
<td>0.063***</td>
</tr>
<tr>
<td></td>
<td>(0.017)</td>
<td>(0.017)</td>
<td>(0.025)</td>
<td>(0.025)</td>
<td>(0.025)</td>
<td>(0.025)</td>
<td>(0.025)</td>
<td>(0.025)</td>
</tr>
<tr>
<td>fam_wealth</td>
<td>0.001***</td>
<td>0.001***</td>
<td>0.001***</td>
<td>0.001***</td>
<td>0.001***</td>
<td>0.001***</td>
<td>0.001***</td>
<td>0.001***</td>
</tr>
<tr>
<td></td>
<td>(0.0001)</td>
<td>(0.0001)</td>
<td>(0.0001)</td>
<td>(0.0001)</td>
<td>(0.0001)</td>
<td>(0.0001)</td>
<td>(0.0001)</td>
<td>(0.0001)</td>
</tr>
<tr>
<td>college_res_p</td>
<td>0.169***</td>
<td>0.183***</td>
<td>0.169***</td>
<td>0.183***</td>
<td>0.169***</td>
<td>0.183***</td>
<td>0.169***</td>
<td>0.183***</td>
</tr>
<tr>
<td></td>
<td>(0.035)</td>
<td>(0.034)</td>
<td>(0.035)</td>
<td>(0.034)</td>
<td>(0.035)</td>
<td>(0.034)</td>
<td>(0.035)</td>
<td>(0.034)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.286***</td>
<td>0.216***</td>
<td>0.006</td>
<td>−0.060</td>
<td>−0.333***</td>
<td>−0.338***</td>
<td>−0.266**</td>
<td>−0.345***</td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.013)</td>
<td>(0.046)</td>
<td>(0.045)</td>
<td>(0.071)</td>
<td>(0.070)</td>
<td>(0.107)</td>
<td>(0.106)</td>
</tr>
<tr>
<td>Observations</td>
<td>1,867</td>
<td>1,867</td>
<td>1,867</td>
<td>1,867</td>
<td>1,867</td>
<td>1,867</td>
<td>1,867</td>
<td>1,867</td>
</tr>
<tr>
<td>R²</td>
<td>0.033</td>
<td>0.053</td>
<td>0.054</td>
<td>0.074</td>
<td>0.160</td>
<td>0.168</td>
<td>0.165</td>
<td>0.186</td>
</tr>
<tr>
<td>Adjusted R²</td>
<td>0.033</td>
<td>0.053</td>
<td>0.052</td>
<td>0.072</td>
<td>0.157</td>
<td>0.166</td>
<td>0.161</td>
<td>0.182</td>
</tr>
<tr>
<td>Residual Std. Error</td>
<td>0.452 (df = 1865)</td>
<td>0.447 (df = 1865)</td>
<td>0.448 (df = 1863)</td>
<td>0.443 (df = 1863)</td>
<td>0.421 (df = 1629)</td>
<td>0.419 (df = 1629)</td>
<td>0.438 (df = 1132)</td>
<td>0.432 (df = 1132)</td>
</tr>
</tbody>
</table>

Note: *p<0.1; **p<0.05; ***p<0.01

Table 12: Linear probability model
Table 2 shows the results of a logit model. As in Table 12, all specifications show that the amount of transfers are statistically significant.

**B Variance de composition in Dynamic Programming**

In a general problem, the present discounted value of utility can be written as

\[ B = \sum_{t=0}^{T} \beta^t u(x_t) \]  \hfill (3)

where \( x_t \) is the state at time \( t \), which may follow a Markov process. The value function is defined as the sum of expected discounted future payoffs, given the initial state \( x \)

\[ J(x) = \mathbb{E}(B|x_0 = x) \]  \hfill (4)

It is useful to define the following operator

\[ M(x) = \mathbb{E}(B^2|x_0 = x) \]  \hfill (5)

Notice that \( J(x) \) and \( M(x) \) can be written as

\[ J(x) = u(x) + \beta P(x'|x)J(x') \]  \hfill (6)
\[ M(x) = u(x)^2 + 2\beta u(x)P(x'|x) + \beta^2 P(x'|x)M(x') \]  \hfill (7)

The previous two equations can be computed recursively. Using \( J(x) \) and \( M(x) \) we can compute the variance decomposition of a variable

\[ \text{Var}(Y|x) = \mathbb{E}(Y^2|x) - \mathbb{E}(Y|x)^2 = M(x) - J(x)^2 \]  \hfill (8)

To compute the variance decomposition, I use

\[ \text{Var}(Y) = \mathbb{E}(\text{Var}(Y|x)) + \text{Var}(\mathbb{E}(Y|x)) \]  \hfill (9)

The first term on the right hand side are computed using \( M(x) \) and \( J(x) \) and the initial distribution over states \( g(a, h, \theta) \). The second term on the right hand side is computed using the value function and the initial distribution over states.
C Numerical Solution

Grids I define grids for ability $\theta$, human capital $h$ and financial capital $a$. The model is solve using parallel computation in the $\theta$ dimension.

Function approximation Policy and value functions are solved using collocation methods. I solved the model in the collocation nodes and I interpolate between nodes. I use linear interpolation.

Optimization problem The problem is solved using backward induction. At each $t$, the optimization problem is solved using golden search method for one dimensional problem (e.g. during retirement), using a two-dimensional golden method for two-dimensional problems (e.g. during working periods) and a nested golden search for $s$ and a bivariate numerical solver for savings and transfer (to solve for inter-vivos transfers). This last piece is by far the most challenging and computationally expensive part of the algorithm.

Distributions Given the discretization of the state space, the policy functions and exogenous shocks define transition matrices over states. The steady state of the model is computed as the ergodic distribution of the transition matrices of the model.