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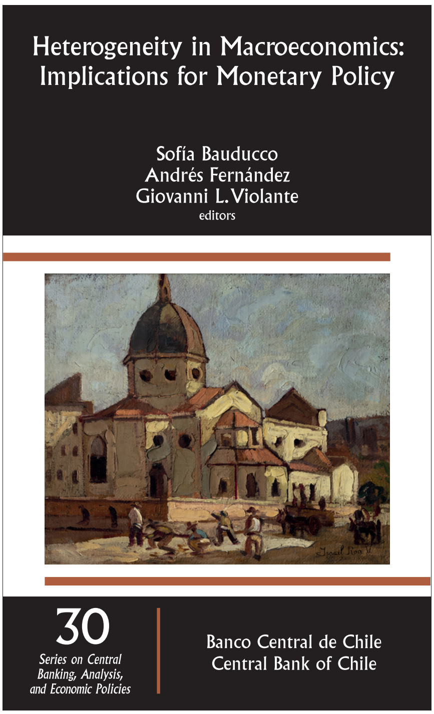

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Published in Santiago, Chile
by the Central Bank of Chile
Manufactured in Chile

ISBN (print) 978-956-7421-74-9
ISBN (digital) 978-956-7421-73-2
Intellectual Property Registration 2024-A-10537

ISSN 0717-6686 (Series on Central Banking, Analysis, and Economic Policies)

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During the Covid-19 pandemic, the Chilean government provided unprecedented economic assistance to households. Direct fiscal transfers through stimulus checks amounted to nine percent of the country's GDP. Aditionally, three times during the period, policymakers allowed for the possibility of withdrawing up to ten percent of the workers' individual pension accounts savings. This policy provided households with access to additional resources equivalent to 19 percent of GDP. Overall, the extra liquidity provided amounted to 28 percent of GDP, thus becoming Chile's most extensive support package in recent history.[1]

The magnitude of these measures highlights how important it is to understand the impact of fiscal transfers on economic activity. However,

The opinions and mistakes are our exclusive responsibility and do not necessarily represent the opinion of the Central Bank of Chile or its board. We thank Gastón Navarro and attendees at the XXV Annual Conference of the Central Bank of Chile for fruitful comments and Giancarlo Acevedo, Javiera Azócar, Ignacio Rojas, and Valentina Vásquez for superb research assistance.

1. To put these numbers in context, before the Covid-19 pandemic, the Chilean government's total spending in subsidies and direct transfers—including education and health—was about 11% of GDP.

Heterogeneity in Macroeconomics: Implications for Monetary Policy, edited by Sofía Bauducco, Andrés Fernández, and Giovanni L. Violante, Santiago, Chile.

our paper does not specifically focus on the Covid-19 experience.[2] Instead, we aim to make a more general point by emphasizing the significance of policy design progressivity in achieving the expected effects on aggregate outcomes, building on the findings of Céspedes and others (2013).

Throughout our analyzed sample period, from 2018 to 2022, we document significant heterogeneity in the scope and progressivity of twelve programs. This heterogeneity allows us to study the differential impact on macroeconomic outcomes of policies with different degrees of progressivity. We start by empirically studying the macroeconomic effects of fiscal transfers. First, we estimate a Bayesian structural vector autoregressive model (BSVAR) to show that fiscal transfers significantly impact economic activity. Second, we document that some policies were mainly flat along the income distribution, while others displayed significant progressivity, thus showing how households with different marginal propensities to consume (MPCs) were affected by the transfers varied across time and policies. Third, with the help of micro data on credit- and debit-card transactions at the municipal level,[3] we study whether fiscal transfers with different degrees of progressivity showed a differentiated impact on household card purchases. To do so, we estimate a local projection-like equation of the dynamic effects of different policies and find that, while all of them show significant effects on this proxy for consumption, the impact of progressive transfers was significantly larger than their nonprogressive counterparts. In other words, these results show that, per unit of help, progressive fiscal transfers, by stimulating purchases the most, were more effective in increasing aggregate consumption. These results support the view that the Chilean economy displays strong non-Ricardian elements, which motivate the use of models that depart from the permanent income hypothesis.

To study to what extent (and under what conditions) transfers progressivity has a role at the aggregate level, we build an heterogeneous agents New Keynesian (HANK) model for the Chilean

2. Vaskov and others (2022) present a comprehensive analysis of the macroeconomic effects of the different fiscal programs implemented by the Chilean government during the Covid-19 pandemic.

3. Administratively, Chile is subdivided into 346 municipalities, also called communes. Wikipedia defines them as "the smallest administrative subdivision in Chile. It may contain cities, towns, villages, hamlets, and rural areas. A conurbation may be broken into several communes in highly populated areas, such as Santiago, Valparaíso, and Concepción." See https: //en.wikipedia.org/wiki/Communes_of_Chile

economy featuring progressive and nonprogressive transfer policies. Both policies are modeled as lump-sum transfers to households. Our model follows Auclert and others (2018), who develop a general equilibrium model with heterogeneous agents and nominal rigidities to study the macroeconomic effects of fiscal policy in the United States. We extend their analysis by considering two features we find essential for the case of Chile: unemployment—with search and matching (SAM) frictions—and progressivity of fiscal transfers. The model also features capital adjustment costs and a government that can finance its spending through taxes and debt accumulation.

Following a strategy similar to Kaplan and others' (2018), we calibrate the model to the Chilean economy by matching the share of hand-to-mouth (HtM) households as documented in household wealth surveys. We also use highly granular administrative data (from the Social Security Administration) on labor income quarterly to calibrate the household's income risk and consumption profiles.

To fix ideas, we propose a statistic we dub the "policy slack", that summarizes to what extent the policy undertaken is expansionary. We define the policy slack as the excess transfer delivered to households due to fluctuations in income. For instance, a positive slack in a downturn means transfers are more generous than needed to offset the household's income loss. We show that a positive policy slack is present in some of the policies implemented in Chile. Furthermore, the slack is heterogeneous across different households and policies. We also show that, under certain conditions, we can summarize the effects of policies on consumption by the relationship between the slack and the households' MPCs. In particular, we decompose the fluctuations in consumption into an average effect, which summarizes how averages fluctuate, and a distributional effect, which summarizes how the distribution of the slack affects the evolution of aggregate consumption. Moreover, we show that the distributional component is significant for all calibrations. It then follows that, when evaluating the effects of fiscal policies, it is crucial to consider not only the magnitude of the policy itself but also how far the policy took each household away from their 'normal' income. We then show that the progressivity of the transfers considerably affects the macroeconomic impact of the programs in that the more concentrated on high MPCs they are, the higher the response of aggregate variables. This result is especially marked when the government finances its spending with debt instead of taxes, so tax-paying households do not contemporaneously pay the additional government expenses. We also find that the aggregate effect (as it is common in this literature) depends on how investment responds. We find, however, that this dependence is mostly orthogonal to the progressivity of the policy. Therefore, it does not affect the differential impact between high and low progressivity transfers. Furthermore, we show that more progressive transfers, as they affect the economy more, have more substantial general equilibrium effects than the less progressive ones.

Related Literature. A relevant part of the HANK literature emphasizes the role of fiscal policy and how it relates to non-Ricardian agents in the economy. Oh and Reis (2012) study the role of targeted transfers in the context of the Great Recession of 2008–2009, and point out the need for models that account for the positive effects of transfers; McKay and Reis (2016) study the role of progressive fiscal policies to show quantitatively that unemployment benefits and progressive taxes generate an attenuation of the business cycle because of their role as automatic stabilizers; Ferriere and Navarro (2020) study the role of tax progressivity for the transmission of government spending, and show that in times where spending is progressively financed, the fiscal multiplier was higher in the U.S. than in times where taxes were less progressive; Hagedorn and others (2019) dissect the transmission of government spending and transfers into the aggregate economy in HANK models; Auclert and others (2018) show that HANK models feature a Keynesian multiplier that gives rise to a Keynesian cross that amplifies the effects of fiscal policies; Kaplan and Violante (2018) argue that HANK models feature stronger nonequivalence than their representative agent counterparts, showing that the inclusion of heterogeneous agents changes both the transmission mechanism and the aggregate effect of fiscal shocks. This paper also relates to the literature on HANK with SAM frictions. We closely follow Gornemann and others (2016), who study the role of SAM in the transmission of monetary policy with heterogeneous agents, and Ravn and Sterk (2020), who show analytically how HANK and SAM frictions interact.

Finally, this paper is related to the empirical analysis of the effects of fiscal transfers on consumption. We follow Johnson and others (2006) and Parker and others (2013), who study the effects of the 2001 and 2008 fiscal rebates on consumption to estimate MPCs in the U.S. by using the Consumer Expenditure Survey. Another relevant paper is Misra and Surico (2014), who estimate the heterogeneous effects of these rebates. We study the dynamic effects of fiscal transfers as in a local projection analysis following the literature on the estimation of MPCs.

We contribute to this literature in four dimensions. First, we show suggestive evidence that the progressivity of transfers matters for the transmission of these policies, i.e., more progressive transfers have stronger effects on aggregates. Second, we extend the theoretical analysis to the labor market to study how unemployment affects the transmission mechanisms of fiscal transfers.[4] Third, we show that the effects of policies can be decomposed into an average effect and a distributional effect (extending Patterson, 2019), and that the way the policy is distributed across households with different MPCs is crucial. Finally, we show that a relevant part of the transfers' second-round general equilibrium effects is driven by the presence of frictional unemployment.

The remainder of the paper is organized as follows. Section 1 presents the empirical evidence we use to motivate this paper. Section 2 describes the model. Section 3 discusses the calibration. Section 4 describes what we call the policy slack—a statistic that summarizes the expected effect of the shocks on aggregate consumption. Section 5 shows the quantitative results from the model. Section 6 concludes.

1. Fiscal Support Measures in Chile: Stylized Facts and Macroeconomic Implications

In this section, we document some stylized facts about the magnitude and implementation of the fiscal transfers given to Chilean households between 2018 and 2022 and perform some empirical estimations showing the macroeconomic impact of the policies. We start by showing some key macroeconomic aggregates to contextualize the scope of the implemented policies. Then, we describe the amounts involved, both in aggregate and by quintiles of the income distribution. Finally, we show suggestive evidence that the effects these measures have on household expenditure are statistically and economically significant and related to the progressivity of the transfers, motivating our further study on the theoretical channels that may generate the observed heterogeneous impact of the different policies on macroeconomic aggregates.

4. Guerra-Salas and others (2021) emphasize the importance of including unemployment in the analysis of the dynamics of the Chilean business cycle, where variation along the extensive margin of the labor supply is particularly relevant.

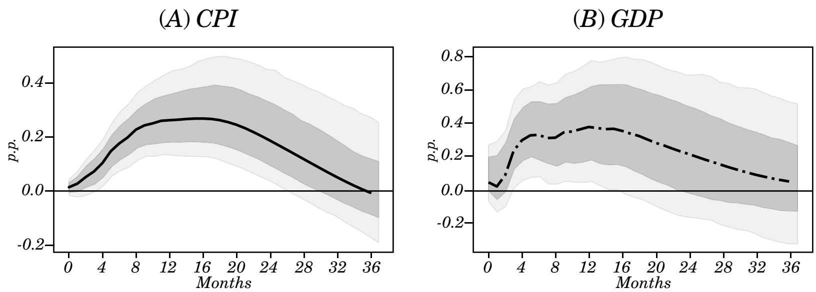
1.1 Fiscal Transfers Stimulate Economic Activity

To study how fiscal transfers affect macroeconomic aggregates, we update the estimates from Céspedes and others (2013) by running a fiscal structural VAR at monthly frequency. We follow Blanchard and Perotti (2002) approach by using a Cholesky identification. Due to our short sample, we estimate the VAR with Bayesian methods. The BSVAR includes fiscal transfers from aggregate fiscal accounts (as a share of GDP), fiscal income, CPI, and industrial production—in that order. The sample spans from January 2005 to August 2022. We consider twelve lags, detrend and seasonally adjust the series, and assume the usual normal-Wishart priors.

The impulse response functions from figure 1 show the response of the log of CPI and the log of GDP to a one-percent of GDP increase in transfers and subsidies, with the corresponding 90 and 68 percent confidence intervals. The results are both statistically and economically significant: a one-percent increase in the transfers-to-GDP ratio generates a 0.4 percent increase in GDP.

Notice that government transfers amounted to about ten percent of GDP during the Covid-19 pandemic, a greater order of magnitude than the exercise in figure 1 so that the effects of the policies undertaken during the crisis would have a substantial impact on the aggregates. This evidence suggests an important non-Ricardian component in the Chilean economy, showing that, as households see their disposable income increase after receiving fiscal transfers, they spend a significant part of this inflow in the subsequent periods, and this leads to substantial short-run effects on industrial production. Also, there is a significant rise in CPI inflation after these shocks.

Figure 1. CPI and GDP Response to a One-Percent of GDP Rise in Government Transfers



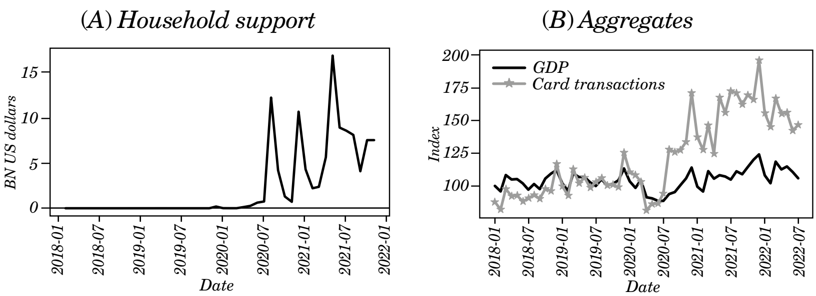
Source: Authors' calculations.

1.2 Not All Support Is the Same

In this section, we characterize the household support measures implemented in Chile in the form of direct transfers from January 2018 to November 2021 and study to what extent these policies affected consumption differently depending on the level of progressivity they displayed.

We consider twelve programs featuring different sizes, timings, cyclicality, and progressivity.[5] The data on the different programs come from the Ministry of Social Security and from the Pensions System Regulator. While these data are available at the individual level, for the empirical analysis performed in this section, we aggregate them at a municipal level as this allows us to draw a direct comparison with our measure for consumption, only available up to that level of aggregation. Figure 2 shows the programs' size and its relationship with economic activity. The left panel depicts the total amount of additional liquidity households obtain thanks to these measures. The right panel, on the other hand, shows how the path of these policies correlated with the evolution of aggregate demand during the period.

Figure 2. Total Household Support and Aggregate Outcomes



Source: Authors' calculations.

5. The twelve programs are i. Family help check; ii. Family base check; iii. Christmas Covid-19 check; iv. School homework check; v. Child homework check; vi. Covid-19 emergency check; vii. Protection check; viii. Emergency Income Covid-19; ix. Emergency Covid-19 2020; x. Guaranteed Minimum Income; xi. Universal Covid-19 check; xii. Pension Funds Withdrawals. In this paper, we consider the latter as fiscal transfers, since pension funds in Chile are fully illiquid accounts in the short run, hence, they are most likely perceived as extra income.

Although all of the features mentioned earlier may play a role in the effectiveness of the different programs, in what follows, we concentrate on only one dimension—the progressivity of the policies. To do so, we define progressivity (conceptually) as the way the government distributes these transfers among households of different incomes. To compute each policy's progressivity, we use the ratio between the absolute amount of liquidity provided to the first and fifth quintiles (Q_1/Q_5). Then, a unitary value for our progressivity score means that all quintiles receive the same amount. That is the relevant threshold since, in the model below, we define MPC as the response of households to a unitary increase in income where this additional amount is the same for everyone.[6] To build the index, we start by classifying each municipality into an income distribution quintile. We then build a per quintile population-weighted transfer measure for all twelve policies and then compute the ratio Q_1/Q_5 for every period for each policy. Finally, we assign each of the twelve programs into two categories: progressive and nonprogressive. As the same program may have different progressivity scores at different periods, we label a program as progressive if the policy has $Q_1/Q_5 > 1$ every month during its implementation and nonprogressive otherwise.

Figure 3 shows the evolution of the average progressivity of both types of policies. We can see that progressivity levels have been falling steadily since early 2020, suggesting a shift towards high-coverage fiscal transfers.[7]

We now analyze the differentiated impact of progressive and nonprogressive policies on consumption. In particular, we study the effect of the policies per unit of additional liquidity provided to the households. To carry out the analysis, we use several data sources, including data on credit- and debit-card transactions at the municipal level as a proxy of consumption obtained from Transbank, a private firm that processes most of the credit and debit transactions in Chile; data on labor income at the municipal level as a control (to account for heterogeneous fluctuations in income) obtained from the Chilean Unemployment Insurance Administration Agency; per municipality

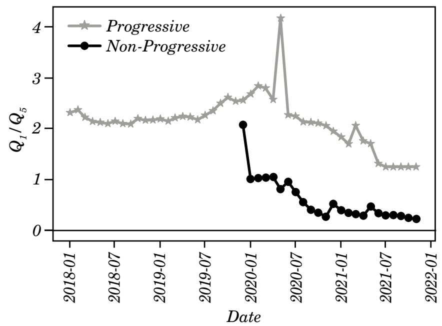
6. This is an absolute measure of progressivity, as opposed to alternative relative progressivity metrics that consider transfers as a share of the household's income or how the transfer helped increase the income of different households. Moreover, a policy with progressivity index 1 (same lump-sum transfer for everyone) is, in fact, progressive in relative terms.

7. In the Appendix, Figure 15 shows the progressivity scores for all of the analyzed programs.

total amounts given by the different programs obtained from the Ministry of Social Security and the Pensions System Regulator; finally, as additional controls, we use data on GDP, CPI, and exchange rates available from the Central Bank and the National Statistics Institute.

Our credit- and debit-card transaction data are available at the municipal level and distinguish between in-person and online purchases. We use the former, as the latter is harder to associate with the buyer's residence. Using these data as a proxy for aggregate consumption has a few shortcomings. First, it only considers card transactions and hence only represents a fraction of the aggregate consumption in the economy, not including cash purchases. Second, although we have access to the firm and place where the transactions were made, we do not know the individual who made the purchase. Due to these restrictions, we carry out our analysis at the municipal level.[8] In a companion paper,[9] we show that card transactions track national accounts data well and that municipalities in Chile are a good approximation of their inhabitants.

Figure 3. Progressivity of Household Support

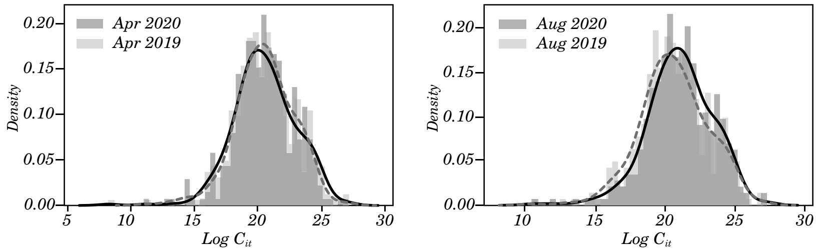


Source: Authors' calculations.

8. The geographical approach is used, for instance, by Mian and Sufi (2009) and Mian and others (2013) to study the effects of wealth on consumption. This approach is also extensively discussed by Guren and others (2020) to disentangle general equilibrium from the partial equilibrium effects of these estimates.

9. García and others (2023b).

Figure 4. Histograms of Consumption at Municipal Level, Selected Dates



Source: Authors' calculations.
Note: Solid line: 2020. Dashed line: 2019.

The aggregation of the more granular fiscal support data down to the municipality level is, as mentioned above, a compromise due to the availability of consumption data. Still, its level of aggregation is appropriate for our analysis, given the observed heterogeneity across municipalities in all the dimensions we are studying: consumption, income, and fiscal support. Figure 4 helps us visualize this by showing the cross-sectional distribution of consumption at the municipal level on selected dates. The figure allows us to point out some relevant facts. First, there is considerable heterogeneity with significant dispersion. Second, the distributions are not static, as they seem to evolve: In April 2020, we observed a tightening of the distribution with respect to 2019; perhaps even more importantly, we observed a rightwards shift in consumption in August 2020, the date of the first pension funds withdrawal, where households received a significant liquidity influx. An outlier does not drive that month's shift, as we observe that in almost all municipalities, consumption rose. These facts give us confidence that aggregating at a municipal level allows for a good representation of the heterogeneity we want to exploit in our analysis.

We study the differential effects of progressive and nonprogressive policies by exploiting the abovementioned heterogeneity. We follow the specification by Misra and Surico (2014), who estimate the effects of the 2001 and 2008 rebates in the United States by using the Consumer Expenditure Survey.[10] To be able to analyze not only

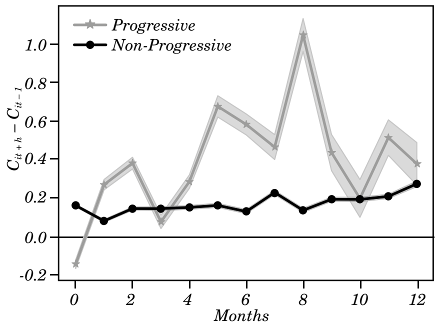
10. Misra and Surico (2014) further study the heterogeneous effects of those rebates following Johnson and others (2006) and Parker and others (2013). A similar approach is also used by Fuster and others (2020), who use surveys from experiments to study the effects on consumption of raising households' income.

the contemporaneous response of consumption to fiscal transfer shocks but also their dynamics, we estimate the following local projection-like regression:

$$C_{it+k} - C_{it-1} = \alpha_k + \beta_k T_{it}^{p} + \delta_k T_{it}^{np} + \Gamma_k^{'} X_{it} + \epsilon_i + \psi_t + \varepsilon_{ikt}, \text{ for } k = 0, \dots, K \quad (1)$$

where C_{it+k} is total credit- and debit-card purchases in municipality i in period $t+k$; α_k is a constant for projection k; T_{it}^{p} and T_{it}^{pn} denote the total amount of progressive and nonprogressive policies given to a municipality i in time t; ϵ_i and ψ_t are respectively a municipality and a time fixed-effect; and X_{it} is a vector of controls that include two lags of income growth and of a mobility index at a municipal level, as well as two lags of T_{it}^{p}, and T_{it}^{pn}.[11] The estimated coefficients β_k and δ_k denote the consumption response up to period $t+k$ after the household support given in period t.[12]

Figure 5. Response of Consumption to Progressive and Nonprogressive Policies



Source: Authors' calculations.

11. We control for an index of mobility which varies along municipality and over time. In Chile, the lockdowns during Covid-19 were at a municipal level, with their degrees varying from 1 (the most restrictive) to 5 (the least restrictive).

12. Robustness exercises with four and eight lags yield qualitatively similar results.

Figure 5 shows the results from estimating Equation (1). The figure presents the effect on household card spending after both types of policies. Several results are worth commenting on. First, transfers have a positive and significant impact on consumption. The regression results show that municipalities that received more transfers saw a more pronounced increase in their consumption. Second, there is a significant differential effect on consumption between progressive and nonprogressive policies. On the one hand, the peak effect on consumption after a progressive transfer is almost five times higher than after a nonprogressive. However, on the other hand, the response to progressive policies is more front-loaded than nonprogressive policies, appearing much more evenly distributed over time. In the remainder of the paper, with the help of a HANK model, we will study the theoretical reasons behind these results.

There are a few essential points to address. First, the observed consumption responses may only partially reflect the reactions to exogenous fiscal transfers, even when considering factors such as income and employment at the municipal level. This could be due to consumption decisions being influenced by increased transfer expectations. While the short interval between policy announcement and implementation (one month) suggests the possibility of exogeneity, definitive causal claims cannot be made. Second, due to the aggregation of individuals up to the municipal level, these results can be interpreted neither as fully partial equilibrium MPCs (as they include potential spillovers from the municipal aggregate consumption to the individual household) nor fully general equilibrium aggregate effects (as it does not consider the GE effects that an increase in aggregate consumption has on a municipality's consumption spending). This issue, common to all estimates using cross-sectional data, arises due to what is called the missing intercept problem,[13] where we cannot be sure of the total effect of a shock on the aggregate economy, and we can only infer the differential effect of the more exposed individuals versus the average, which in this case would mean the differences in consumption between municipalities that received more transfers than others. Finally, while the debit- and credit-card transaction data closely resemble aggregate consumption patterns, it is important to note that it might not capture total household consumption. Biases could arise due to self-selection in debit-/credit-card usage. For instance, households without prior card usage might adopt it after the transfers, mainly since many programs

13. See Wolf (2023) and Nakamura and Steinsson (2014).

require a bank account for more accessible receipt of financial aid. These biases are assumed to be evenly spread across municipalities and affect both policies equally, allowing an unbiased comparison of differential effects between the two policy types.

2. A HANK Model with Heterogeneity in Transfers Progressivity

To rationalize the facts presented in the previous section and study the different policies' roles, we build a HANK model calibrated for Chile. We closely follow the approach—and methods—presented by Auclert and others (2021). The model is a HANK with unemployment risk[14] with liquid and illiquid assets.[15]

We extend the model to include unemployment risk, as it has been shown that the extensive margin of the labor supply is a fundamental driver of the income risk and employment fluctuations in Chile.[16] This feature is especially relevant for households at the bottom of the distribution and that depend crucially on labor income.

In the model, the government is able to provide transfers in different amounts to households of different income levels. In addition, the government can finance its spending by issuing debt or raising taxes. Finally, the model has the usual features of New Keynesian models: price rigidities, monopolistic competition in intermediates, and capital adjustment costs. Since we use the methods developed by Auclert and others (2021) to solve the model, which relies on economies with aggregate shocks but without uncertainty, we omit the expectation operator over time in the model's description. In particular, the method applies a linearization of the sequence space, which relies on unexpected shocks but with a known future path.

2.1 Households

The economy is populated by a continuum of households of measure one. Households are heterogeneous in their assets, productivity, and employment state. Households deliver utility from consumption and leisure. They maximize the time-separable utility function $\mathbb{E}\left\{\sum_{k=0}^{\infty}\beta^k u\left(c_{t+k},h_{t+k}\right)\right\}$, where $u(c,h)$ is of the usual CRRA

14. As in Ravn and Sterk (2020) or Gornemann and others (2016).
15. As in Auclert and others (2018).
16. See Guerra-Salas and others (2021).

form $\frac{c^{1-\gamma}}{1-\gamma}-\psi\frac{h^{1+\varphi}}{1+\varphi}\mathbb{I}_e$ and $\mathbb{E}$ is the expectation operator over labor productivity and employment uncertainty. $\mathbb{I}_w$ = 1 if working and zero otherwise (extensive margin), and h is hours worked (intensive margin). There are N_z possible idiosyncratic states in the productivity dimension where the probability of transitioning between states z and z' is given by $\Pi(z, z')$.

Agents can be employed or unemployed. If employed, they supply H_t hours common to all workers due to labor market frictions we explain below (thus, $h_t = H_t$ for all households). Workers earn $(1-\tau_t)w_t H_t z_t$, where w_t is the wage per efficient hour and τ_t is a proportional labor income tax. If unemployed, households receive an unemployment benefit denoted by ω, distributed in proportion to agents' productivity times wages $w_t z_t$. Following Diamond-Mortesen-Pissarides' framework, we denote by δ the separation rate and $f(\theta)$ job-finding rate of transitioning between the states w and u such that $s = [w,u]$. Hence, $\Pi(z, z', s, s')$ is the transition matrix considering both unemployment and income risk. Consequently, income becomes $y_t(z_t, s)$ with $y_t(z_t, .) = [(1-\tau_t)\, w_t H_t z_t,\, z_t w_t\, \omega]$.

Agents can trade in two assets, i.e., $\boldsymbol{a} \equiv \{a_1, a_2\}$. These assets pay an interest rate $r_{ht}(h = \{1,2\})$ and are subject to a non-borrowing constraint. The value function of an agent in the states $(z, \boldsymbol{a}, s)$ at time t is, therefore[17]

$$
\begin{aligned}
V_t(z,\boldsymbol{a},s) &= \max_{c,\boldsymbol{a}} u(c) + \beta \sum_{z,s} \Pi(z,z',s,s')\, V_{t+1}(z',\boldsymbol{a}',s') \\
\text{s.t. } c + \sum_h a'_h &= \sum_h (1+r_{ht})\, a_h + y(z,s) + f_t(z) + d_t(z) \\
\boldsymbol{a} &\geq 0.
\end{aligned}
$$

Households receive a fiscal transfer which is a function of household productivity $f_t(z)$; i.e., it depends on the household type. We determine this function in the calibration below. $d_t(z)$ are individual firms' dividends received by households. For the structure of assets of households, we take the approach by Auclert and others (2018), who assume there is a fully liquid (government bonds) and a fully illiquid

17. In Appendix B we present the value functions and first-order conditions of this problem.

asset (capital and equity). The illiquid assets returns are accrued in the liquid account. These assumptions allow us to match the high MPC (through the high share of HtM) and a high level of aggregate wealth while keeping the model tractable.[18]

Given optimal policies $c_t^{\star}(z,\boldsymbol{a},s), a_t^{\prime\star}(z,\boldsymbol{a},s), b_t^{\prime\star}(z,\boldsymbol{a},s)$ and denoting $\Psi(z,\boldsymbol{a},s) = \Pr(z_t = z, a_{t-1} \in A, s_t = s)$ the probability of that combination of states at the start of date t, the distribution Ψ_t has a law of motion

$$\Psi_{t+1}(z',\boldsymbol{a}',s') = \sum_{z,s} \Psi_t(z',\boldsymbol{a}'^{\star-1},s')\Pi(z,z',s,s'), \tag{2}$$

where $\boldsymbol{a}'^{\star-1}$ are the inverse of the optimal policies of $\boldsymbol{a}$. For simplicity, we summarize in an index i, the combination of possible states, i.e., $i = (z, \boldsymbol{a}, s)$. Therefore, in what follows, $\Psi(z, \boldsymbol{a}, s) = \Psi(i)$, and the aggregate of a variable $x_t(i)$ is given by $\int x_t(i)\Psi(i)di = X_t$. However, we use the long notation when needed.

With the distribution and the optimal allocations we compute the aggregates $C_t = \int c_t(i)\Psi(i)di$ and the stock of liquid assets, $B_t = \int b_t(i)\Psi(i)di$ with counterpart in the government budget constraint.

2.2 Government

Fiscal policy is one of the main ingredients in our model. The government, in our setting, allocates its spending between government consumption G_t, fiscal transfers to households $f_t(z)$, and unemployment benefits ω. Transfers are heterogeneous across households and can be progressive ($f_t'(z) < 0$), regressive ($f_t'(z) > 0$), or flat $f_t'(z) = 0$. How transfers are distributed across households satisfies $\int f_t(z)\Psi(i)di = T_t$ where T_t denotes the aggregate amount of transfers. The government finances its spending by issuing real-denominated debt B_t^g and by charging proportional taxes on labor income. Government debt is held by households in their liquid account and pays a real return r_t. Transfers are lump-sum in the sense that households take these as given and do not enter their first-order conditions. However, they affect optimal decisions due to market incompleteness. The government's budget constraint is then given by

$$B_{t+1}^g = T_t + G_t + \omega w_t U_t - \tau_t w_t H_t N_t + (1 + r_t)B_t^g.$$

18. As in Auclert and others (2018), we assume the fact shown by Fagereng and others (2021) that households do not change their illiquid assets in response to income shocks.

The evolution of the fiscal balance depends on a smoothing parameter ρ_T, which determines to what extent additional spending is financed with debt according to:

$$dB_t^g = \rho_T \left(dB_{t-1}^g + dT_t \right).$$

This fiscal balance rule captures the fact that governments do not necessarily raise taxes contemporaneously to finance additional spending, as they can also issue more debt. As we will see below, the government financing strategy is key for characterizing consumption dynamics in response to fiscal transfers in general equilibrium.

2.3 Firms

There is a continuum of identical firms (indexed by $j \in [0,1]$) that produce differentiated goods using capital and labor, combining them with a Cobb-Douglas function $y_{jt} = Z_t k_{jt-1}^{\alpha} n_{jt}^{1-\alpha}$, with Z_t denoting an aggregate productivity level. Although identical, these intermediate firms are in monopolistic competition and set prices taking into account the demand for their variety. Varieties are aggregated with a Dixit-Stiglitz aggregator with a price elasticity equal to $\frac{\mu_p}{\mu_p - 1}$, with μ_p being the steady state markup charged by these firms. Price setting is subject to quadratic Rotemberg adjustment costs, with the cost given by $\Theta_{jt}^{\pi} = \frac{\mu_p}{\mu_p - 1} \frac{1}{2\kappa_p} \left[\log\left(1 + \pi_{jt}\right) \right]^2 y_{jt}$, with κ_p being the adjustment cost parameter that is also the slope of the Phillips curve. Intermediate firms solve:

$$J_t \left(p_{jt-1} \right) = \max_{y_{jt}, p_{jt}, k_{jt}, n_{jt}} \left\{ \frac{p_{jt}}{p_t} y_{jt} - w_t h_{jt} n_{jt} - r_t^k k_{jt-1} - \Theta_{jt}^{\pi} + \frac{J\left(p_{jt}\right)}{1 + r_{t+1}^a} \right\}$$

s.t.

$$y_{jt} = Z_t k_{jt-1}^{\alpha} \left(h_{jt} n_{jt} \right)^{1-\alpha},$$

$$y_{jt} = \left(\frac{p_{jt}}{p_t} \right)^{-\frac{\mu_p}{\mu_p - 1}} Y_t.$$

The first-order conditions, after symmetry, read

$$\log\left(1+\pi_t\right) = \kappa_p\left(mc_t - \frac{1}{\mu_p}\right) + \frac{1}{1+r^a_{t+1}}\frac{Y_{t+1}}{Y_t}\log\left(1+\pi_{t+1}\right)$$

$$mpl_t = \left(1-\alpha\right)mc_t\frac{Y_t}{N_t}$$

$$r^k_t = \alpha mc_t\frac{Y_t}{K_{t-1}},$$

where mc_t is the marginal cost. The aggregate amount of profits generated each period by intermediate firms is given by

$$\Pi^y_t = \left(1-mc_t\right)Y_t - \Theta^\pi_t.$$

2.4 Labor Markets

There is a union that determines hours worked (the intensive margin) by aggregating households' preferences, solving the individual problem at an aggregate level. This maximization procedure generates the following labor supply, which is given by the average marginal rate of substitution equal wages:

$$\psi H^\varphi_t = \mathcal{U}' w_t,$$

with $\mathcal{U}' = \int\left(1-\tau\left(z_t\right)\right)z_t u'\left(c_t\left(i\right)\right)\Psi_t\left(z,a,s=e\right)di.$

To account for fluctuations in unemployment and unemployment risk, we consider a labor market with search frictions as in Ravn and Sterk (2020) and Gornemann and others (2016). The model is a canonical Diamond-Mortensen-Pissarides model. We assume there is a Cobb-Douglas matching function $M\left(U_t,V_t\right) = m_t U^\gamma_t V^{1-\gamma}_t$, which leads to a job-finding probability $f_t\left(\theta_t\right) = m_t\theta^{1-\gamma}_t$ and a job-filling probability $q\left(\theta_t\right) = m_t\theta^{-\gamma}_t$, where $\theta_t = \frac{V_t}{U_t}$ is the market tightness. U_t is the measure of unemployed workers with $U_t = \int d\Psi\left(z_t,b,a,s=u\right)$, and the level of employment is given by $N_t = 1 - U_t$. The probability of becoming unemployed while working is given by an exogenous separation probability δ.

We assume that households cannot individually supply—and set—labor. Instead, there is an intermediary for each type who hires and sells labor services. This firm's value of a worker with productivity z_t is

$$J(z_t) = (mpl_t - w_t)z_t + (1-\delta)\frac{1}{1+r_{t+1}}\mathbb{E}_z\left[J\left(z_{t+1}|z_t\right)\right],$$

where mpl_t is the marginal product of labor. The free-entry condition for these intermediaries is

$$\frac{c_v}{q(\theta_t)} = \frac{1}{1+r_{t+1}}\int_{z_t}\mathbb{E}_z\left[J\left(z_{t+1}|z_t\right)\right]d\Phi(z_t, b, a, s = u).$$

Additionally, we use a Nash-inspired wage rule

$$w_t = (1-\eta)\omega + \eta(mpl_t + c_v\theta_t),$$

where η is workers' wage bargaining power.

Finally, the intermediary generates profits from the difference between the marginal productivity of labor and the real wage given by

$$\Pi_t^w = mpl_t - w_t.$$

These profits are delivered to households in the same way monopolistic profits are.

2.5 Capital

We assume there is an investment fund that produces capital. The investment fund owns the economy's capital stock K_t. The fund makes the economy's investment decision subject to an adjustment cost $\Gamma_t(K_{t+1}, K_t)$, solving the problem

$$\max_{K_{s+1}, I_s}\sum_{s=0}^{\infty}\left(\frac{1}{1+r_s}\right)\left[r_t^k K_t - I_t - \Gamma(K_{s+1}, K_s)\right]$$

s.t.

$$K_{s+1} = (1-\delta_K)K_s + I_s,$$

where $\Gamma(K_t, K_{t+1}) = \frac{1}{2\delta_K\epsilon_I}\left(\frac{K_{t+1}-K_t}{K_t}\right)^2 K_t$. The first-order conditions are:

$$\left(1+r_{t+1}\right)q_t^k = r_{t+1}^k - \left[\frac{K_{t+1}}{K_t} - \left(1-\delta_K\right) + \frac{1}{\delta_K \epsilon_I}\left(\frac{K_{t+1}-K_t}{K_t}\right)^2\right] + \frac{K_{t+1}}{K_t} q_{t+1}^k$$

$$q_t^k = 1 + \frac{1}{\delta_K \epsilon_I}\left(\frac{K_{t+1}-K_t}{K_t}\right),$$

equations that reduce to the Tobin's-Q solution.

2.6 Dividends

Dividends in this economy are given by the sum of the return to capital, profits from intermediate producers, and profits from the labor intermediary. Therefore, it can be shown that dividends are given by

$$\text{Div}_t = Y_t - w_t H_t N_t - \Theta_t^\pi - c_v V_t - I_t - \Gamma\left(K_{t+1}, K_t\right).$$

These dividends are delivered with an ad-hoc rule similar to Kaplan and others (2018), in proportion to household productivity.

2.7 Monetary Authority

In the presence of nominal rigidities, the real interest rate r_t is determined by monetary policy, which sets the nominal interest rate i_t according to a Taylor rule that responds to inflation and unemployment:

$$i_t = i^* + \phi_\pi\left(\pi_t - \overline{\pi}\right) + \phi_U \frac{\left(U_t - U\right)}{U}.$$

We denote by $\phi_\pi > 0$ and $\phi_U < 0$ the preference parameters for inflation and unemployment respectively. Monetary authorities seek a nominal interest rate target in steady state given by i^*. Given inflation and the nominal interest rate, the real rate is determined by the Fisher equation $\left(1+r_t\right) = \frac{\left(1+i_t\right)}{1+\pi_{t+1}}$.

2.8 Aggregation

Total consumption expenditure is given by

$$C_t = \int c(i)\Psi(i)\,di. \tag{4}$$

Goods market clearing implies

$$Y_t = C_t + I_t + G_t + \Theta_t^{\pi} + c_v V_t + \Gamma\left(K_{t+1}, K_t\right),$$

and the market for bonds closes:

$$B_t^g = \int b\Psi(i)\,di.$$

3. Calibration

3.1 Households

Households' Assets. We follow Kaplan and others (2018) to develop our aggregated two-asset (liquid-illiquid) structure. For this purpose, we use a mix of data from the Chilean Financial Regulator (CMF) and the Chilean Household Financial Survey (EFH). This latter survey is the Chilean counterpart of the Survey of Consumer Finances. We consider this mix to have a reasonable estimate of the aggregates (from CMF) and the distribution of assets in the Chilean economy.

We closely follow the taxonomy proposed by Kaplan and Violante (2014), which is given by the following components (summarized in table 1). On the side of liquid assets, revolving debt corresponds to bank credit cards, lines of credit, bank or financial consumer loans, credit cards from nonbanking institutions, consumer loans in commercial houses (cash advances), credits in savings banks, cooperatives, educational loans, and other nonmortgage debts. Deposits are the total amount households keep in their checking or sight accounts. We also include equity in the liquid account from the data, which is the sum of investment in shares, mutual funds, participation in investment funds, and investment in other equity instruments (options, futures, swaps, among others). Finally, fixed income is the total amount households have invested in different instruments such as time deposits, bonds, savings accounts, and insurance with savings.

We consider three illiquid assets: net housing, defined as the value households assign to their primary home or other real estate they own, discounting the present value of the mortgage loan debt; net durables which correspond to the value of automotive assets, such as cars or trucks, motorcycles, vans or utility vehicles, and other motorized vehicles (boats, planes, helicopters, etc.), as well as other assets such as agricultural or industrial machinery, animals, works of art, etc., discounted from the debt in auto loans.

Table 1 summarizes this taxonomy as a fraction of the 2017 annual GDP. When considering aggregates, we obtain figures not so far from the ones shown in Kaplan and others (2018) for the United States. Liquid assets are a small fraction of total wealth, and housing is the largest fraction of wealth. This means that in Chile is also appropriate to use the liquid-illiquid split when considering the assets' structure.

Regarding the shape of the distribution of assets, we use the EFH to build these distributions. However, unlike Kaplan and others (2018), we only focus on the share of HtM of Chilean households, which is a key target in our calibration. Table 2 shows the shares of HtM of Chilean households. We define an HtM household as one that holds up to five percent of their quarterly income in liquid assets (in absolute value). We find that for Chile, the total share of HtM is about 39 percent of households. This figure is considerably higher than that of the United States, which is about 30 percent. Another difference that we find with respect to the U.S. is that in Chile the share of wealthy HtM households is 31 percent, while in the U.S it is six percent. The poor's HtM, though, is 8 percent, i.e., lower than the 20 percent the U.S. has. These differences are interesting, but in this paper, we only use the total share of HtM to calibrate our model.[19]

Table 1. Taxonomy of Households' Assets in Chile in 2017. Values as a Percentage of GDP

Liquid (B)		*Iliquid (A)*		*Total*
Revolving consumer debt	-0.12	Net housing	1.93	
Deposits	0.05	Net durables	0.13	
Fixed income	0.12			
Equity	0.12			
Total	0.17		2.06	2.23

Source: Commission for the Financial Market (CMF) and Internal Revenue Service.

Table 2. Share of Wealthy and Poor Hand-to-Mouth Households (Relative to the Total Population)

	Data	
Poor	Frac. With $B \approx 0$ and $A = 0$	0.08
Wealthy	Frac. With $B \approx 0$ and $A > 0$	0.31

Source: Authors' calculations.

19. We study the effects of these features for Chile in García and others (2024).

Income distribution and income risk. Empirically, the challenge in estimating the frequency of earnings is that almost all high-quality panel earnings data are available only at an annual (or lower) frequency. We overcome this issue by employing a confidential dataset from the Chilean Pension Regulator.[20] We calculate the empirical moments of the distribution of income fluctuations to obtain a discretized process for income risk. In particular, following Guvenen and others (2019), we consider fluctuations in income at different frequencies. We consider from the second to the fourth standardized moments (variance, skewness, and kurtosis), which, as has been shown in previous literature,[21] can be essential for aggregate fluctuations and wealth accumulation.

We assume idiosyncratic income (in logs) is given by the sum of two processes z_{1t} and z_{2t}:

$$y_t = z_{1t} + z_{2t}, \tag{5}$$

where z_{it} follows

$$z_{it} = \rho_i z_{it-1} + \sigma_i \varepsilon_{it}$$

$$\varepsilon_{it} = \begin{cases} \mu_{it} \geq p_i \sim \mathcal{N}(0,1) \\ \qquad \mu_{it} < p_i \end{cases}$$

$$\mu_{it} \sim U[0,1].$$

Therefore, we estimate parameters $\{\rho_1, \rho_2, \sigma_1, \sigma_2, p_1, p_2\}$. As noted by the previous literature, the combination of these two processes returns high kurtosis (given by a $p_i \neq 0$) and can match the moments of the growth in income at lower frequencies.

To match the moments of the empirical distribution with the income process in Equation (5), we approximate z_1 and z_2 using a discretization method first proposed by Farmer and Toda (2017) and Tanaka and Toda (2013, 2015). This method is based on matching conditional moments of the discrete approximation with the moments of the true continuous-state process. This is similar to the Rouwenhorst method proposed by Kopecky and Suen (2010), extended for nonlinear, non-Gaussian Markovian processes. Therefore, our job is to pin down the parameters that describe the processes z_i, namely ρ_i, σ_i, p_i to

20. See appendix A for a description of this database.
21. See Kaplan and others, 2018 and McKay (2017).

match the moments observed in the data and then apply the method by Farmer and Toda (2017) to obtain the discretized version that we feed into the model. We find the parameters by minimizing a loss function that takes a proposed set of parameters and computes how far we are from the desired moments.

Table 3 shows the moments of quarterly labor income for one-quarter and twenty-quarters log-change in labor income and the variance of the log of income ($\log(y_t)$). We compare the empirical moments with the ones we obtain with our discretization method.[22] What we observe here is that, naturally, the variance increases with the lag of the difference, and these distributions have a high kurtosis, which decreases with the lag of the change. Although decreasing, the kurtosis is still higher than that of a normal distribution for the twenty-period change. Table 3 shows that our model matches the empirical moments well.

We show the estimated process in table 4. We estimate a permanent process with high persistence with a half-life of around 43 years (a career shock) and a low probability of occurrence: workers receive these shocks every 3.5 years. The other shock is less persistent but more likely. Households receive it almost every quarter, while its half-life is about 0.4 quarters. With these parameters, we build the transition matrix to discretize these processes, and we consider three points for the persistent component and eleven for the transitory component.[23]

Table 3. Empirical and Estimated Moments of Labor Earnings in Chile at a Quarterly Frequency

Moment	*Data*	*Model*
Var log (y_t)	0.719	0.714
Var Δlog (y_t)	0.195	0.226
Var Δ_{20}log (y_t)	0.463	0.448
Kur $\Delta_{\log}$ (y_t)	11.589	11.617
Kur Δ_{20}log (y_t)	6.143	6.076

Source: Unemployment Fund Administration, Chile.

22. In García and others (2024), we study the role of all these features in Chile. In particular, we compare Chile's moments to those observed in the United States. We show that Chile has a higher variance than the United States but a lower risk.

23. This process suggests that in Chile, income risk is higher than what we observe in the United States. A reason for this high risk is the high worker turnover in Chile. Albagli and others (2017) conclude that, turnover in Chile is higher than all of the OECD countries.

Table 4. Parameter Estimates for Idiosyncratic Income Process

ρ_1	ρ_2	σ_1	σ_2	p_1	p_2
0.996	0.145	0.511	0.382	0.071	0.958

Source: Authors' calculations.

3.2 Labor Markets and Firms

Labor Markets. We use the same targets as in the quantitative model of the Central Bank of Chile.[24] We calibrate unemployment in steady state at eight percent, the vacancy filling probability $q(\theta) = 0.8$, and the separation rate to $\delta = 0.04$. In steady state, the job-finding probability is given by

$$u = \frac{\delta}{\delta + p(\theta)} \Rightarrow p(\theta) = \delta \cdot \frac{1-u}{u} = 0.46.$$

The Nash bargaining parameter is set to $\eta = 0.5$.[25] We set $\alpha = 0.5$ (Hosios condition). We calibrate the productivity of the matching function to satisfy the previous conditions, with $m = \frac{p(\theta)}{\theta^{1-\alpha}}$. Finally, we set the Frisch elasticity of labor supply $1/\varphi$ equal to one, and we calibrate the parameter of disutility of labor to match $H_t = 1$.

Firms. We assume in the steady state a capital level of 2.01 as a share of GDP (8.04 quarterly) to match the value of illiquid assets in steady state in table 1. The capital share α_k is equal to 1/3. Productivity Z in steady state is set to have GDP in steady state equal to one ($Y = 1$). The depreciation rate is equal to 0.01,[26] and in the baseline calibration, the capital adjustment cost parameter is set to $\epsilon_I = 2$. Finally, we assume markups are $\mu_p = 1.1$, and the slope of the price Phillips curve is set to 0.1.

3.3 Government

We set the Taylor rule parameters to $\phi_\pi = 1.25$ and $\phi_U = -1$ in the baseline calibration. We set the level of government spending and fiscal transfers equal to ten percent of GDP each. Fiscal transfers

24. García and others (2019).
25. As in García and others (2019) and Mortensen and Pissarides (1994).
26. From García and others (2019).

have two components, a progressive and a nonprogressive transfer. We set both to five percent of GDP. Individual transfers are defined by a nonlinear function $f(z) = T_t z^{-\aleph_f} f_0$, where f_0 is a scalar which ensures $\int f(z)\Psi(i)di = T_t$ and $\aleph_f$ is the level of progressivity. We solve the model with two transfers which only differ in the progressivity level $\aleph_f$. In the next sections, we introduce two types of policies simultaneously, a progressive and a nonprogressive one, to match the distribution of two selected policies delivered in 2020. These parameters are $\aleph_p = -1.1$ $\aleph_{np} = 0.4$ in the progressive and the nonprogressive policies respectively. We explain how we set these parameters in the next section. Finally, we set the tax rate on dividends equal to 25 percent, and we show results for different ways of government financing, ρ_T.

3.4 Solution Method

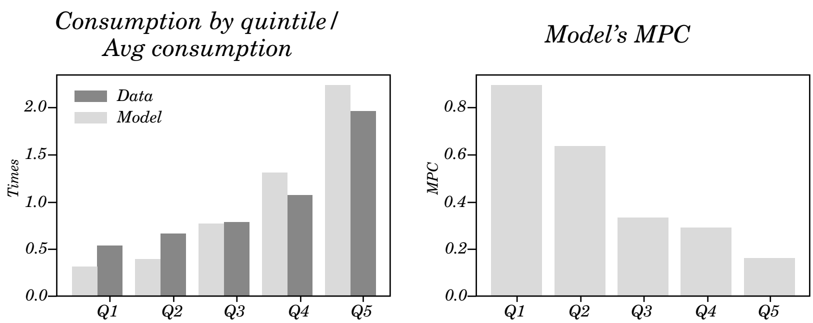
To solve this heterogeneous-agent model with borrowing constraints, we follow Auclert and others (2021). To solve the value function we use Carroll's (2006) endogenous grid method, which is a fast and accurate algorithm to solve these kinds of problems. Then, we use a Newton method to solve the steady state of this economy. And finally, to solve the model with aggregate shocks, we follow Auclert and others (2021) as well, who propose to write the model in its sequence space and linearize around that system of equations. The method relies on the fact that any model without aggregate uncertainty can be written as a sequence of equations in the transition. This is, if we assume shocks are one-time and unexpected, we can write the system as a sequence of equations in the transitional dynamics. This system of equations, which is given by $T \times M$, with T standing for the horizon of the transition and M the number of equations to solve, can be linearized around the steady state. This linearization leads to jacobians of all variables with respect to others, and the impulse-responses can be obtained by a composition of these jacobians. This method, based on Boppart and others (2018), is faster, more accurate, and more robust than methods like the ones that follow Reiter (2009). We refer the reader to the paper for more details on the method.

3.5 Calibration in the Steady-State and Micro Fit

To solve the steady state we leave free the disutility of labor (ψ), the discount factor (β), the level of labor income taxes (τ_w), the vacancy cost (c_v), and aggregate bond holdings (B or B^g). As targets, we set an interest rate of five percent yearly, a HtM share of 0.39, hours at one; the unemployment rate is determined implicitly by satisfying the free-entry condition in the labor market, and τ^w by satisfying the government budget constraint. After this calibration procedure, we obtain $\beta = 0.95$, $\psi = 0.51$, $c_v = 0.19$ which leads to 0.8 percent of GDP in vacancy costs, a tax rate equal to $\tau^w = 0.08$, and aggregate bond holdings equal to 0.18 as a share of annual GDP. Finally, we set the elasticity of intertemporal substitution equal to one ($\gamma = 1$).

Additionally, our goal is to characterize the consumption distribution well. In figure 6 we show the distribution of consumption in steady state and this distribution in the data. The left-hand panel shows consumption with respect to the mean by quintile in the model and the data. Our calibration overestimates inequality: in the model, consumption of the first quintile is lower than the data, and consumption of the fifth quintile is larger. This may be problematic if we are interested in inequality itself. However, as we are interested in the response of each quintile, and there is a fall in MPCs along the income distribution, we argue that this feature of our model underestimates the effects of progressivity. That, because a more progressive policy gives money to households with high MPCs, which in our model are weighted lower than in the data. This is, if the distribution of consumption was as in the data, the response to progressive transfers would be larger than in our results. The right-hand panel shows the MPCs by quintile in the income distribution. In our calibration, the MPCs are decreasing in the quintile of income. The consumption-weighted MPC in our model is 0.31 quarterly. These values are larger than in the US,[27] as expected.

Figure 6. Distribution of Consumption and MPCs



Source: Authors' calculations.

27. As reported by Kaplan and others (2018), 0.16.

4. The Policy Slack

Often, fiscal transfers occur in response to exogenous aggregate shocks affecting households' income. When fiscal support is larger than the drop in household income, there is a gap we call the policy slack, which for household i we denote by $\chi_t(i)$ and satisfies the following identity:

$$dT_t(i) = d\chi_t(i) - dy_t(i) \tag{6}$$

with $dT_t(i)$ being the change in transfer and $dy_t(i)$ the change in income of household i. Equation (6) means that the policy slack is a measure of extra resources taken or given to a household with respect to a perfect compensation to the fall in income, where this perfect compensation is the response of transfers that keeps consumption of most consumers constant at their steady-state levels.

The policy slack is empirically observable. Take, for example, the policies undertaken during the Covid-19 pandemic. It was a combination of progressive and nonprogressive programs with different policy slacks. Table 5 shows how both policies allocated resources differently for each quintile of the income distribution. In this case, the more progressive policies showed a markedly decreasing pattern along the income distribution: the fifth quintile received less than one percent of their income, whereas the first quintile received close to 20 percent. A second group of less progressive policies was much less targeted towards low-income households. In those programs, high-income households received about the same as low-income households as a share of income. Transfers were one of many sources of policy slack. Also to be considered is the drop in income, which is also very heterogeneous across households. While the first quintile saw their income fall by about 19 percent, the income of a typical household from the fifth quintile remained practically unchanged. The combination of fiscal programs and Covid-related drops in income resulted in very heterogeneous policy slacks across quintiles. Due to the relatively low amounts given by the average progressive policy, it generated a negative policy slack. On the other hand, the more generous nonprogressive ones generated an overcompensation in the income fall.

Table 5. Household Support Measures in 2020

Quintile	$\frac{T_t^p(q)}{y_t(q)}$	$\frac{T_t^{np}(q)}{y_t(q)}$	$dy_t(q)$	$d\chi_t^p(q)$	$d\chi_t^{np}(q)$	$d\chi_t^{\text{tot}}(q)$
Q1	0.20	0.25	-0.19	0.01	0.06	0.26
Q2	0.09	0.31	-0.24	-0.15	0.07	0.16
Q3	0.04	0.32	-0.27	-0.23	0.05	0.09
Q4	0.02	0.28	-0.19	-0.17	0.11	0.11
Q5	0.003	0.24	0.00	0.003	0.24	0.243

Source: Authors' calculations.
Notes: Total annual labor income by quintile (q) in 2019 $y_t(q)$ obtained from the Social Security Administration (AFC), $T_t^p(q)$ is total progressive transfers by quintile in 2020, and $T_t^{np}(q)$ are nonprogressive transfers by quintile in 2020. $d(y_t(q))$ denotes the change in income of households at a given quintile (q) between 2020 and 2019.

This policy slack can be an important statistic because it helps us to evaluate the policies and has a direct effect on consumption. Moreover, in models with inequality, not only does the size of the policy slack matter, but also its distribution, which is directly related to the progressivity of the policy and interacts with the MPCs of households. To explain this, denote the household's i MPCs with $M_{t,s}(i)$, which is the response of consumption in t to an income windfall on s with $s = [0,\ldots,T-1]$. Therefore, a matrix $M_t(i)$ summarizes the intertemporal MPCs and is a $T \times T$ matrix for every i where each row is the response in period t to a shock in period s. Hence, the response of household consumption in t is the multiplication of the $M_t(i)$, the row of the matrix $M(i)$ for the period t, and the whole path of future policy slacks $d\chi(i)$, with $d\chi(i)$ being a column vector. Hence, the response of consumption in period t, assuming a constant interest rate, is

$$dC_t = \int M_t(i) d\chi(i) di, \tag{7}$$

which can be rewritten as

$$dC_t = \underbrace{\bar{M}_t \, d\bar{\chi}}_{\text{Average Effects}} + \underbrace{COV_i\left(M_t(i), d\chi(i)\right)}_{\text{Distributional Effects}}. \tag{8}$$

Equation (8) decomposes consumption fluctuations into two components: the average effect and the distributional effect of the policy slack. The first component represents the responses to the size of the policy, and the second one represents the response of consumption to the progressivity of the policy by the relationship between households' MPCs $M_t(i)$ and the policy slack $\chi(i)$. This implies that given the same average MPCs and a given path in the policy slack, there are effects from how fluctuations in income and transfers are distributed among households. These decompositions have recently been popular in the HANK literature.[28]

We can decompose consumption further by separating 'direct' effects from the slack and 'indirect' effects[29] to analyze if the covariance object fluctuates more from partial or general equilibrium effects:

$$dC_t = \underbrace{\overline{M}_t\, d\overline{T} + COV_i\left(\overline{M}_t(i), \overline{d}T(i)\right)}_{\text{Direct}} + \underbrace{\overline{M}_t\, d\overline{y} + COV_i\left(M_t(i), dy(i)\right)}_{\text{Indirect}}. \quad (9)$$

Next, we apply this decomposition to the calibrated model. To do so, we first solve the model in the baseline calibration, assuming a constant real interest rate and calibrating the progressivity of the policy to match the second and third columns of table 5, which requires $\aleph_p = 0.4$ and $\aleph_{np} = -1.1$. After solving the model, we compute the paths for the average and distributional effect of a one percent of GDP increase in transfers (with a persistence of 0.5). In this case, we show the results for the decomposition of consumption in figure 7. Consumption increases in response to both shocks. However, the progressive transfer is twice as effective as the nonprogressive.

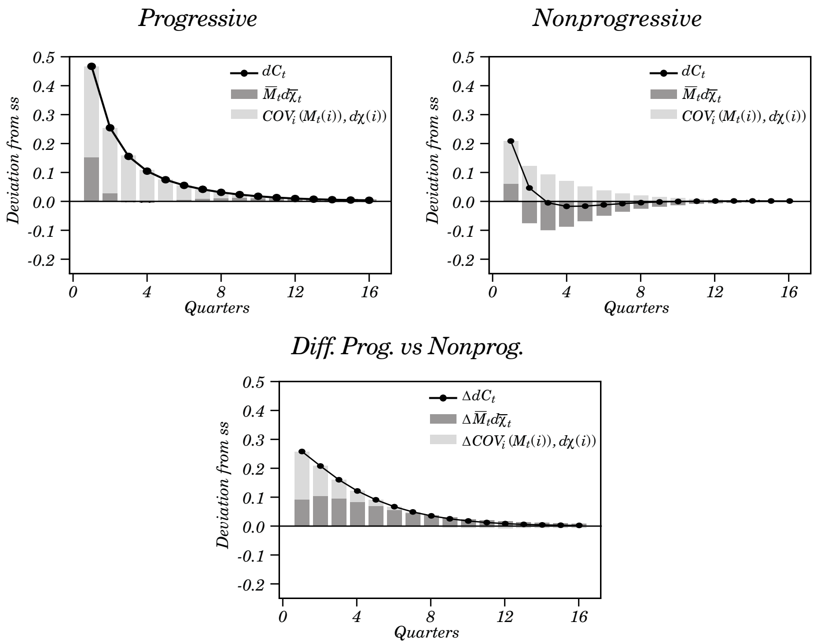
We find that the progressive policy propagates through both channels in the whole horizon. This is, the progressive policy is able to generate a positive response through the average and the distributional channels. However, this is not the case in the nonprogressive policy, where the bigger share of the fiscal transfers given to the wealthier households leads to an average channel that partially reverses the effects generated from redistribution in general equilibrium.

In figure 8, we show the decomposition described in Equation (9), separating both the distributional and the average components into their direct and indirect effects. Since MPCs and the path for the transfer are the same in both cases, the differences arise from the covariances and the general equilibrium effects.

Figure 8 shows different effects on consumption from progressive and nonprogressive transfers. In the former, the component COV $(M_t(i), dT_t(i))$ is positive, contributing to the increase in consumption. In the latter, however, the component COV $(M_t(i), dT_t(i))$ is negative and hence, counteracts the initial impulse of the transfer.

28. See Patterson (2019).
29. As in Kaplan and others (2018) or Auclert (2019).

Figure 7. Consumption Decomposition in Average and Distributional Effects. Constant r and $\rho_T = 0.5$

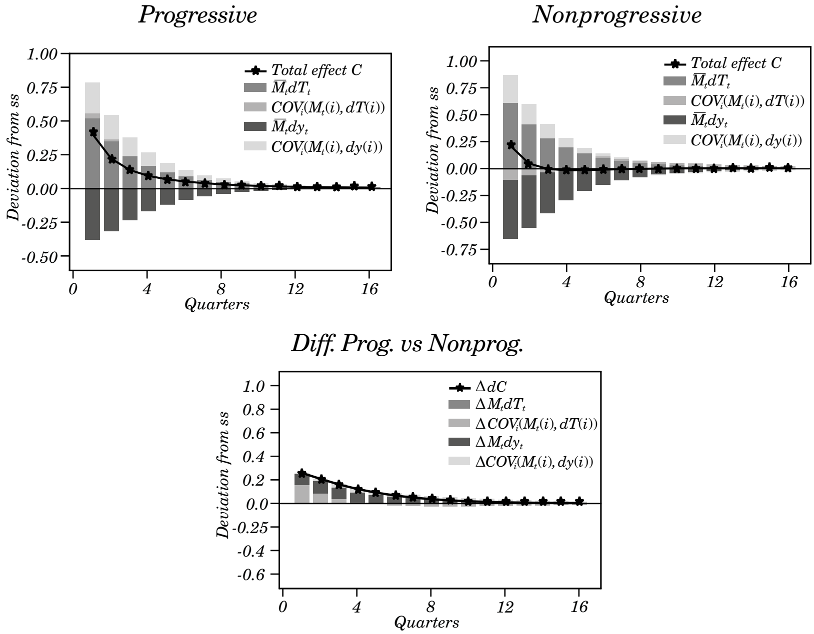


Source: Authors' calculations.

On top of those effects, we have the general equilibrium effects (or indirect effects) through fluctuations in income. For our calibration, we find that the average effect is negative for both policies but stronger for the nonprogressive policy. Moreover, these indirect effects seem to be distributed unevenly among the distribution of MPCs: the covariance term associated with that channel is positive, counteracting the negative response of the average. This result is due mainly to the countercyclical dividends our model features, which is the main driver of the negative responses in the average indirect effect. Since we distribute these dividends increasingly in productivity (and hence, on MPCs), we observe a positive COV $(M_t(i), dy_t(i))$.[30]

30. Aldunate and others (2023) find that labor income in the lowest quintiles responds more strongly than in the highest quintiles to foreign shocks, which would generate an additional source of positive COV $(M_t(i), dy_t(i))$ and hence, that would deliver more amplification in our setup.

Figure 8. Consumption Decomposition in Average and Distributional, Indirect, and Direct Effects. Constant *r* and $\rho_T = 0.5$



Source: Authors' calculations.

While in this exercise we study the role of these channels in the response of consumption to fiscal transfers, the decompositions from Equations (8) and (9) can be used to study the effects of a broad range of policies, like the ones described in table 5.

5. Quantitative Analysis

In this section, we explore the aggregate effects of transfers when we relax the assumption of fixed real interest rate and let monetary policy have a more active role over the business cycle. In addition to that, we show the role of government financing rules on the expected effect of the transfers.

In particular, in the exercises that follow, we show the responses of macroeconomic variables to a rise in fiscal transfers of one percent of GDP. We assume a persistence of 0.5, halving the impulse every quarter. For each of the exercises, we show two figures. First, we show the responses to transfers, with the effect of progressive transfers on the top panels and the nonprogressive on the bottom panels. We show the response of macroeconomic aggregates, labor market variables, and prices. Second, we show a decomposition of the policies' effect on consumption by separating the total effect on consumption between 'direct' and 'indirect' effects.[31]

Baseline $\rho_T = 0.5$ and tight monetary policy. Figure 9 shows the response of macroeconomic variables to a rise in fiscal transfers of one percent of GDP. In this case, the 'baseline' monetary policy reacts to inflation and unemployment ($\phi_\pi = 1.25$ and $\phi_U = -1$). This figure shows that, quantitatively, fiscal transfers impact all the macro variables, triggering a boom on impact with a subsequent bust in both cases, progressive and nonprogressive. However, in both cases, transfers have a low total effect on consumption due to the endogenous response of labor income taxes (to finance the transfer partially) and unemployment due to an endogenous response with feedback from consumption and output. Additionally, the endogenous response of the nominal interest rate contributes to the downturn after the shock.

Figure 10 shows, on the other hand, the decomposition of the response of consumption between the direct (that from changes in transfers) and indirect (the other variables). The direct and indirect effects are different. In particular, the direct effect in the progressive case is about 40 percent on impact more significant than in the nonprogressive. Consistent with the evidence in the previous section, the indirect effect becomes more negative in the nonprogressive than the progressive. This latter result is significant because it is evidence of the transfer's large impact and that general equilibrium effects operate in the transfer's transmission.

Loose monetary policy and $\rho_T = 0.5$. Figure 11 shows the same exercise but in a case where the monetary authority does not respond to inflation or unemployment rate. In this case, we assume monetary policy 'coordinates' with the fiscal policy in stimulating the economy by not responding to the fiscal impulse. The consumption response in the progressive case is about twice as large as in the nonprogressive policy. This result is because, as the nominal interest rate does not adjust, the real interest rate falls (due to the rise in inflation and the Fisher equation). This substantial fall in the real interest rate also mutes the response of the tax rate since there is lower debt servicing

31. As in Kaplan and others (2018).

during these periods. The third reason for a significant consumption surge is the fall in the unemployment rate, which we did not observe in the previous case. This result shows the significant general equilibrium effects of having a progressive transfer, which is paid by itself because taxes go down.[32]

Figure 12 shows the decomposition of consumption into direct and indirect effects. We observe that the indirect effects are significant in both cases. The indirect effect of the progressive policy is larger than that of the less progressive one. These results imply that progressive policies have stronger impacts through targeting high MPCs than nonprogressive ones and through the general equilibrium effects.

Figures 11 and 12 provide evidence that the effect of these kinds of policies depends on the monetary policy stance. Therefore, to maximize the response to government transfers, policies must target households with high MPCs, and monetary policy must be loose. Conversely, when monetary policy counteracts these impulses, fiscal policy may become contractionary. These results are present in any New Keynesian model.[33] Finally, having a monetary policy stance that does not entirely counteract the fiscal impulse is not unrealistic, at least in the short run. We observed this policy coordination in times of Covid-19.

Tight monetary policy and Tax-Financed Transfers, $\rho_T = 0$. Figure 13 shows the previous exercises when government finances transfers with taxes $\rho_T = 0$. Even though the responses to the transfer are lower than in the previous exercises, the differences between progressive and nonprogressive transfer are significant. At least on impact, the response of the progressive case is positive, and the nonprogressive is negative. The response of the progressive one is about 0.4 i.e., 30 percent lower than the partially financed transfers. This result arises from the increases in labor income taxes, unemployment, and real interest rate (due to the rise in inflation).

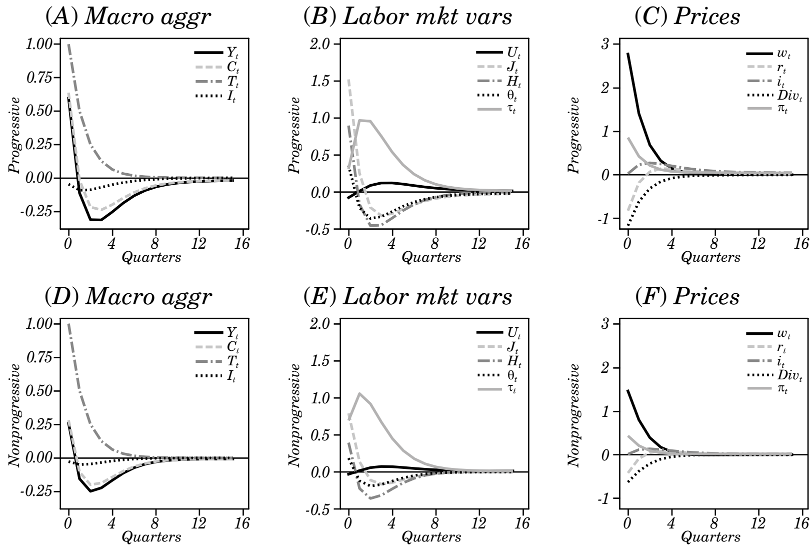
The decomposition in figure 14 shows that, in this case, the indirect effect is negative in both cases, and the direct effect is about the same as the one in the previous cases. However, the general equilibrium effect is less negative for the progressive transfer than for the nonprogressive one.

More Results. In the Appendix, we show other combinations of these exercises. In particular, we find that consumption's response to transfers is the largest in extreme cases of debt-financed transfers

32. This result is also stressed by Angeletos and others (2023).
33. See Woodford (2011).

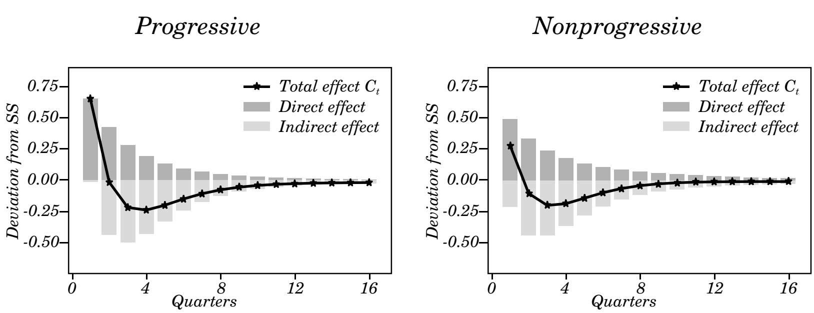
and loose monetary policy. However, in that case, the contribution of progressivity is lower than what we showed above. We also study the effect of muting investment and do not find significant differences between the cases with and without it.

Figure 9. Responses of Aggregate Variables to a 1% Rise in Fiscal Transfers. Tight Monetary Policy and $\rho_T = 0.5$



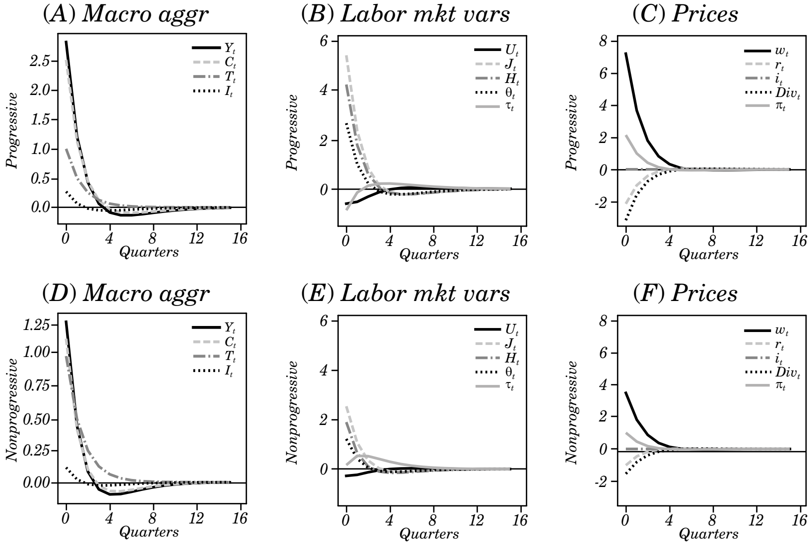
Source: Authors' calculations.

Figure 10. Decomposition of Consumption into Direct and Indirect Effects in Response to Fiscal Transfers. Tight Monetary Policy and $\rho_T = 0.5$



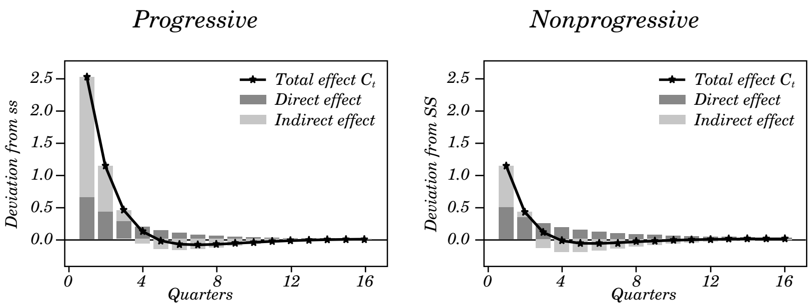
Source: Authors' calculations.

Figure 11. Responses of Aggregate Variables to a 1% Rise in Fiscal Transfers. Loose Monetary Policy and $\rho_T = 0.5$



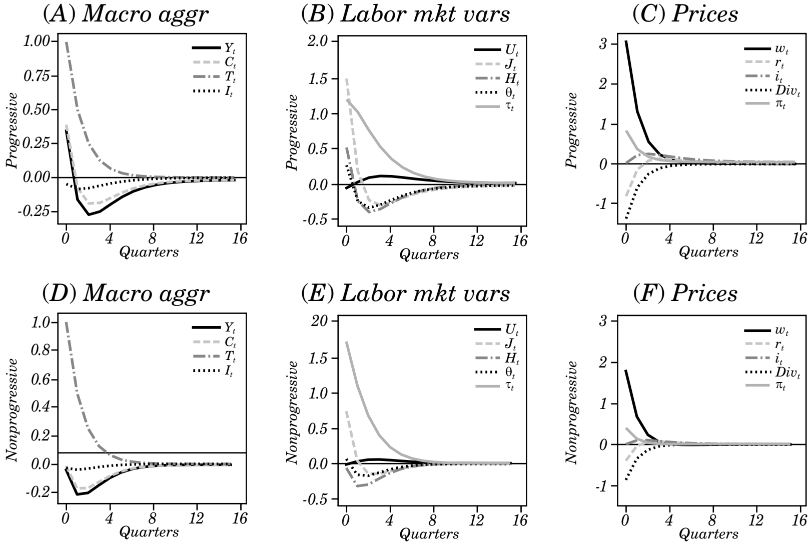
Source: Authors' calculations.

Figure 12. Decomposition of Consumption into Direct and Indirect Effects in Response to Fiscal Transfers. Loose Monetary Policy and $\rho_T = 0.5$



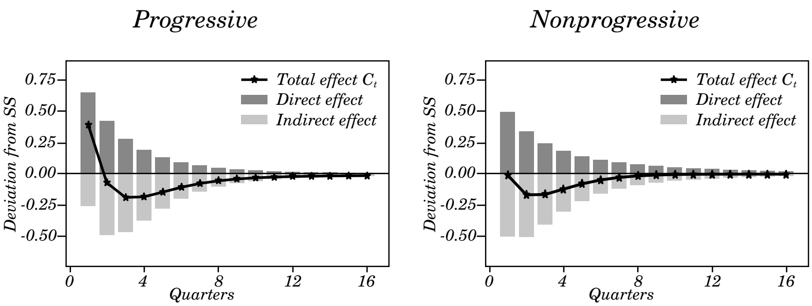
Source: Authors' calculations.

Figure 13. Responses of Aggregate Variables to a 1% Rise in Fiscal Transfers. Tight Monetary Policy and $\rho_T = 0$



Source: Authors' calculations.

Figure 14. Decomposition of Consumption into Direct and Indirect Effects in Response to Fiscal Transfers. Tight Monetary Policy and $\rho_T = 0$



Source: Authors' calculations.

6. Conclusion

In this paper, we build an heterogeneous agents New Keynesian model calibrated for Chile. We test the model implications by comparing its results to empirical facts regarding the effects of fiscal transfers on real activity. These facts derive from two separate estimations. First, fiscal transfers significantly impact GDP and inflation by running a fiscal SVAR as in Blanchard and Perotti (2002).

Second, at a municipal level, we analyze the impact of different fiscal programs between 2018 and 2022. By combining receipts of credit- and debit-card transactions with data on household income and fiscal support, we show that consumption in Chile responds more strongly to policies classified as progressive, suggesting a considerable non-Ricardian behavior of Chilean households.

Our calibrated model can replicate these empirical findings and several other key moments of the Chilean economy. We show that more progressive transfers, associated with higher covariance between allocated funds and household's MPCs, have stronger effects on consumption than less progressive policies. We also show that the magnitude of this differential impact depends crucially on how the government finances its policies and the monetary policy response to the shock. Finally, we decompose the shock's impact between an average and a distributional effect with a statistic that we call the policy slack. We show that a higher transfer progressivity is associated with a higher share of its effect attributed to the distributional channel. A stronger second-round general equilibrium effect compounds the higher direct effect in more progressive policies.

References

Albagli, E., A. Chovar, E. Luttini, C. Madeira, A. Naudon, and M. Tapia. 2023. "Labor Market Flows: Evidence for Chile Using Microdata from Administrative Tax Records." *Latin American Journal of Central Banking*, 4(4): 100102.

Aldunate, R., A. Blanco, A. Fernández, M. Giarda, and G. Navarro. 2023. "The Cross Sectional Labor Market Dynamics After a Foreign Shock." Manuscript, Central Bank of Chile.

Angeletos, G.-M., C. Lian, and C.K. Wolf. 2023. "Can Deficits Finance Themselves?" NBER Working Paper No. 31185.

Auclert, A. 2019. "Monetary Policy and the Redistribution Channel." *American Economic Review* 109(6): 2333–67.

Auclert, A., B. Bardóczy, M. Rognlie, and L. Straub. 2021. "Using the Sequence-Space Jacobian to Solve and Estimate Heterogeneous-Agent Models." *Econometrica* 89(5): 2375–408.

Auclert, A., M. Rognlie, and L. Straub. 2018. "The Intertemporal Keynesian Cross." NBER Working Paper No. 25020.

Blanchard, O. and R. Perotti. 2002. "An Empirical Characterization of the Dynamic Effects of Changes in Government Spending and Taxes on Output." *Quarterly Journal of Economics* 117(4): 1329–68.

Boppart, T., P. Krusell, and K. Mitman. 2018. "Exploiting MIT Shocks in Heterogeneous-Agent Economies: The Impulse Response as a Numerical Derivative." *Journal of Economic Dynamics and Control* 89: 68–92.

Carroll, C.D. 2006. "The Method of Endogenous Gridpoints for Solving Dynamic Stochastic Optimization Problems." *Economics Letters* 91(3): 312–20.

Céspedes, L.F., J. Fornero, and J. Galí. 2013. "Non-Ricardian Aspects of Fiscal Policy in Chile." In *Fiscal Policy and Macroeconomic Performance,* edited by L.F. Céspedes and J. Galí. Santiago, Chile: Central Bank of Chile.

Fagereng, A., M. B. Holm, and G.J. Natvik. 2021. "MPC Heterogeneity and Household Balance Sheets." *American Economic Journal: Macroeconomics* 13(4): 1–54.

Farmer, L.E. and A.A. Toda. 2017. "Discretizing Nonlinear, Non-Gaussian Markov Processes with Exact Conditional Moments." *Quantitative Economics* 8(2): 651–83.

Ferriere, A. and G. Navarro. 2020. "The Heterogeneous Effects of Government Spending: It's All About Taxes." Manuscript, Federal Reserve Board.

Fuster, A., G. Kaplan, and B. Zafar. 2020. "What Would You Do with $500? Spending Responses to Gains, Losses, News, and Loans." *Review of Economic Studies* 88(4): 1760–95.

García, B., S. Guarda, M. Kirchner, and R. TranamiL. 2019. "XMAS: An Extended Model for Analysis and Simulations." Working Papers No. 833, Central Bank of Chile.

García, B., M. Giarda, C. Lizama, and I. Rojas. 2024. "Transmission Mechanisms In HANK: an Application to Chile." *Latin American Journal of Central Banking.*

García, B., M. Giarda, C. Lizama, and D. Romero. 2023. "Time-Varying Expenditure Shares and Macroeconomic Fluctuations." Manuscript, Central Bank of Chile.

Gornemann, N., K. Kuester, and M. Nakajima. 2016. "Doves for the Rich, Hawks for the Poor? Distributional Consequences of Monetary Policy." International Finance Discussion Papers No. 1167, U.S. Board of Governors of the Federal Reserve System.

Guerra-Salas, J., M. Kirchner, and R. Tranamil-Vidal. 2021. "Search Frictions and the Business Cycle in a Small Open Economy DSGE Model." *Review of Economic Dynamics* 39: 258–79.

Guren, A., A. McKay, E. Nakamura, and J. Steinsson. 2020. "What Do We Learn from Cross-Regional Empirical Estimates in Macroeconomics?" NBER Macro Annual No. 35.

Guvenen, F., F. Karahan, O. Serdar, and J. Song. 2019. "What Do Data on Millions of U.S. Workers Reveal about Life-Cycle Earnings Dynamics?" Manuscript, University of Minnesota.

Hadzi-Vaskov, M., E. Luttini, K. Kuester, and L. RicCi. 2022. "Consumption during Covid-19 in Chile." Manuscript, Central Bank of Chile.

Hagedorn, M., I. Manovskii, and K. Mitman. 2019. "The Fiscal Multiplier." NBER Working Paper No. 25571.

Johnson, D.S., J.A. Parker, and N.S. Souleles. 2006. "Household Expenditure and the Income Tax Rebates of 2001." *American Economic Review* 96(5): 1589–610.

Kaplan, G., B. Moll, and G.L. Violante. 2018. "Monetary Policy According to HANK." *American Economic Review* 108(3): 697–743.

Kaplan, G. and G.L. Violante. 2014. "A Model of the Consumption Response to Fiscal Stimulus Payments." *Econometrica* 82(4): 1199–239.

Kaplan, G. and G.L. Violante. 2018. "Microeconomic Heterogeneity and Macroeconomic Shocks." *Journal of Economic Perspectives* 32(3): 167–94.

Kopecky, K.A. and R.M. Suen. 2010. "Finite State Markov-Chain Approximations to Highly Persistent Processes." *Review of Economic Dynamics* 13(3): 701–14.

McKay, A. 2017. "Time-Varying Idiosyncratic Risk and Aggregate Consumption Dynamics." *Journal of Monetary Economics* 88: 1–14.

McKay, A. and R. Reis. 2016. "The Role of Automatic Stabilizers in the U.S. Business Cycle." *Econometrica* 84(1): 141–94.

Mian, A. and A. Sufi. 2009. "The Consequences of Mortgage Credit Expansion: Evidence from the U.S. Mortgage Default Crisis." *Quarterly Journal of Economics* 124: 1449–96.

Mian, A., K. Rao, and A. Sufi. 2013. "Household Balance Sheets, Consumption, and the Economic Slump." *Quarterly Journal of Economics* 128(4): 1687–726.

Misra, K. and P. Surico. 2014. "Consumption, Income Changes, and Heterogeneity: Evidence from Two Fiscal Stimulus Programs." *American Economic Journal: Macroeconomics* 6(4): 84–106.

Mortensen, D.T. and C.A. Pissarides. 1994. "Job Creation and Job Destruction in the Theory of Unemployment." *Review of Economic Studies* 61(3): 397–415.

Nakamura, E. and J. Steinsson. 2014. "Fiscal Stimulus in a Monetary Union: Evidence from US Regions." *American Economic Review* 104(3): 753–92.

Oh, H. and R. Reis. 2012. "Targeted Transfers and the Fiscal Response to the Great Recession." *Journal of Monetary Economics* 59: S50-S64, supplement issue:October 15–16 2010 Research Conference on 'Directions for Macroeconomics: What Did We Learn from the Economic Crises?' Sponsored by the Swiss National Bank.

Parker, J.A., N.S. Souleles, D.S. Johnson, and R. McClelland. 2013. "Consumer Spending and the Economic Stimulus Payments of 2008." *American Economic Review* 103(6): 2530–53.

Patterson, C. 2023. "The Matching Multiplier and the Amplification of Recessions." *American Economic Review* 113(4): 982–1012.

Ravn, M.O. and V. Sterk. 2020. "Macroeconomic Fluctuations with HANK & SAM: An Analytical Approach." *Journal of the European Economic Association* 19(2): 1162–202.

Reiter, M. 2009. "Solving Heterogeneous-Agent Models by Projection and Perturbation." *Journal of Economic Dynamics and Control* 33(3): 649–65.

Tanaka, K. and A.A. Toda. 2013. "Discrete Approximations of Continuous Distributions by Maximum Entropy." *Economics Letters* 118: 445–50.

Tanaka, K. and A.A. Toda. 2015. "Discretizing Distributions with Exact Moments: Error Estimate and Convergence Analysis." *SIAM Journal on Numerical Analysis* 53(5): 2158–77.

Wolf, C.K. 2023. "The Missing Intercept: A Demand Equivalence Approach." *American Economic Review* 113(8): 2232–69.

Woodford, M. 2011. "Simple Analytics of the Government Expenditure Multiplier." *American Economic Journal: Macroeconomics* 3(1): 1–35.

Appendices

Appendix A. Data on Fiscal Aid and Pension Fund Withdrawals

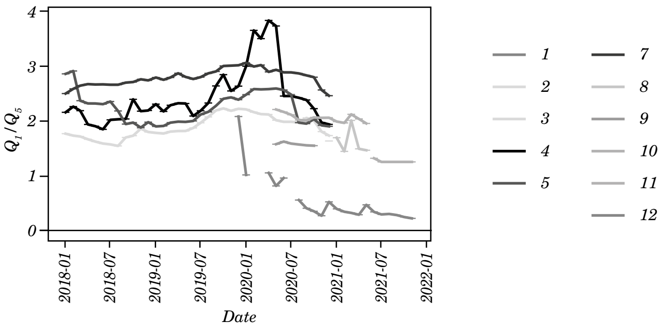
The data on pension fund withdrawals are obtained from the regulator of pension funds. The database is administrative, and we have access to the universe of withdrawals. The database includes the dates of the delivery of the withdrawal, the amount, and an individual identification number.

Until October 2022 there were 11,108,917 requests on the first withdrawal. The average disbursed is 1,422,919 (close to USD 1500). In dollars, the total given amounts to 16.14 billion. The second withdrawal had 9,310,312 requests. The average disbursed was 1460955 pesos (about USD 1500 as well) and the total amounted to USD 13.81 billion. The third withdrawal had 8,866,610 requests in which the average was about USD 1500 as well. The total amount in the third withdrawal was USD 13.05 billion. Therefore, the total amount in withdrawals was USD 43 billion.[34]

The transfer programs available for this study are of different types, sizes, and progressivities. These programs usually target different types of households, focused mainly on poorer ones. We list them as follows: 1. Family help check; 2. Family base check; 3. Christmas Covid-19 check; 4. School homework check; 5. Child homework check; 6. Covid-19 emergency check; 7. Protection check; 8. Covid-19 emergency income; 9. Covid-19 2020 emergency; 10. Guaranteed minimum income; 11. Universal Covid-19 check. These policies have been available since January 2018. These are all direct transfers to individuals, which may be conditional (like homework checks) and unconditional, like Universal Covid-19 checks. These are all targeted to households somehow, as we can observe in figure A.1.

34. Source: Chile' Superintendency of Pensions.

Figure A1. Progressivity of Household Support



Source: Authors' calculations.
Notes: 1. Family help check; 2. Family base check; 3. Christmas Covid-19 check; 4. School homework check; 5. Child homework check; 6. Covid-19 emergency check; 7. Protection check; 8. Covid-19 emergency Income; 9. Covid-19 2020 emergency; 10. Guaranteed minimum income; 11. Universal Covid-19 check; 12. Pension Funds Withdrawals. We exclude policy 6 from the graph because it goes off the chart.

Appendix B. Household Problem

$$V\left(u_t, z_t, b_{t-1}\right) = \max_{c_t, b_t} u\left(c_t\right) + \beta\left[p\left(\theta_t\right)V\left(e_{t+1}, z_{t+1}, b_t\right) + \left(1 - p\left(\theta_t\right)\right)V\left(u_{t+1}, z_{t+1}, b_t\right)\right]$$

$$\text{s.t. } c_t + b_t = (1 + r_t)\, b_{t-1} + \omega z_t - \tau_t \tau(z_t) + d_t d(z_t)\; b_t \geq 0$$

$$V\left(e_t, z_t, b_{t-1}\right) = \max_{c_t, b_t} u\left(c_t\right) + \beta\left[\left(1 - \delta\right)V\left(e_{t+1}, z_{t+1}, b_t\right) + \delta V\left(u_{t+1}, z_{t+1}, b_t\right)\right]$$

$$\text{s.t. } c_t + b_t = \left(1 + r_t\right) b_{t-1} + w_t z_t - \tau_t \tau\left(z_t\right) + d_t d\left(z_t\right)\; b_t \geq 0.$$